# Factorization in DIS at large x

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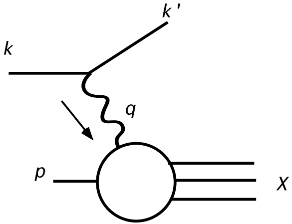
# Introduction

- Factorization and threshold resummation for DIS at $x \to 1$ are usually studied with “diagrammatic” techniques
- SCET has the potential to organize factorization proofs more transparently
- People disagree on how this is done (8 papers)

  (Manohar ’03, ’05; BP ’05, Chay and Kim ’05; Chen, Idilbi, Ji ’05, ’06; Becher, Neubert, BP ’06; Idilbi and Mehen ’07)

Will discuss the approach in Becher, Neubert, BP

# Kinematics and momentum scales in DIS




$$q^2 = -Q^2$$

$$x \equiv \frac{Q^2}{2P\cdot q}$$

- Nucleon: $P^2 = m^2 \sim \Lambda^2_{\mathrm{QCD}}$
- Final State:

$$M_x^2 \approx \frac{Q^2(1-x)}{x}$$

Generic $x$: $\quad Q^2 \sim M_x^2 \gg \Lambda^2_{\mathrm{QCD}}$

Large $x$: $\quad Q^2 \gg M_x^2 \gg \Lambda^2_{\mathrm{QCD}}$

# Factorization at generic $x$

$$F_2^{\mathrm{ns}}(x, Q^2) = \int_x^1 d\xi \, C\left(\frac{\xi}{x}, Q^2, \mu\right) \phi_q(\xi, \mu)$$

Straightforward for generic $x$: $\Lambda_{\mathrm{QCD}}^2 \ll M_x^2 \sim Q^2$

- $C$ is calculated as an expansion in $\alpha_s(Q \sim M_X)$
- IR physics in $\phi_q$ (parton distribution function)

# Factorization at large $x$ ($M_X^2 \ll Q^2$)

$$F_2^{\mathrm{ns}}(x, Q^2) = H(Q^2, \mu) \int_x^1 d\xi \, J\left(Q^2 \frac{\xi - x}{x}, \mu\right) \phi_q(\xi, \mu)$$

- $H$ is a “hard function” depending on hard scale $Q^2$
- $J$ is a “jet function” depending on jet scale $M_X^2$
- $\phi_q$ is the parton distribution function in $\xi \to 1$ limit

**Also need resummation (Thomas Becher’s talk)**

# Where do we stand?

**Generic x:**

$$F_2 = C \otimes \phi_q$$

**Large x:**

$$F_2 = (H \cdot J) \otimes \phi_q$$

Mostly agree that large and generic $x$ are not so different:

- $(H \cdot J)$ is $C$ expanded and subfactorized in large $x$ limit
- $\phi_q$ is standard PDF in large $x$ limit

Not much agreement in:

- The exact structure of IR SCET
- SCET description of PDF

# Overview

1. Demonstrate factorization of the form $C \otimes \phi_q$ at large $x$ diagrammatically to one loop
2. Discuss the subfactorization $C \to H \cdot J$ in SCET
3. Discuss connection between large and generic $x$ in SCET and also power corrections

# Hadronic tensor and $F_2$

QCD effects contained in hadronic tensor:

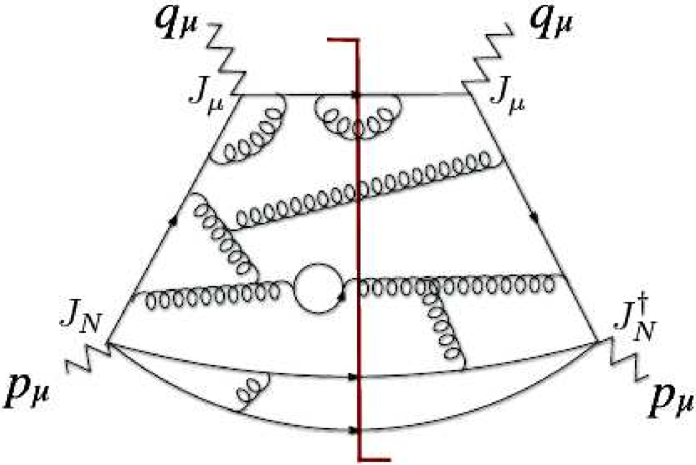
$$
\begin{aligned}
W^{\mu\nu}(p,q) &= i\int d^4x\, e^{iq\cdot x}\, \langle N(p)|\, T\{J^{\dagger\mu}_{\mathrm{EM}}(x)\, J^{\nu}_{\mathrm{EM}}(0)\}\, |N(p)\rangle \\
&= \left(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu}\right) W_1 + \ldots
\end{aligned}
$$

Structure function:

$$
F_2^{\mathrm{ns}}(x,Q^2) = \frac{1}{\pi}\mathrm{Im}\sum_q e_q^2\, x\, W_1(x,Q^2)
$$

“Non-singlet” component obtained by averaging over different nucleons

# Hadronic tensor in QCD




Expand around limit $p^2 \ll Q^2(1-x) \ll Q^2$

- Can't evaluate, try to factorize

# Factorization in DIS

To factorize the hadronic matrix element

$$W^{\mu\nu} = \langle N(P)|T^{\mu\nu}|N(P)\rangle$$

1) Instead evaluate partonic matrix element

$$W^{\mu\nu}_{\text{parton}} = \langle q(p_p)|T^{\mu\nu}|q(p_p)\rangle$$

2) If the partonic structure function sastifies (to all orders)

$$F_{2,parton} = C \otimes \phi_{q,parton}$$

**assume** this is true for the QCD structure function as well

$$F_2 = C \otimes \phi_q$$

# Factorization at one loop

Define coefficient functions through a subtraction:

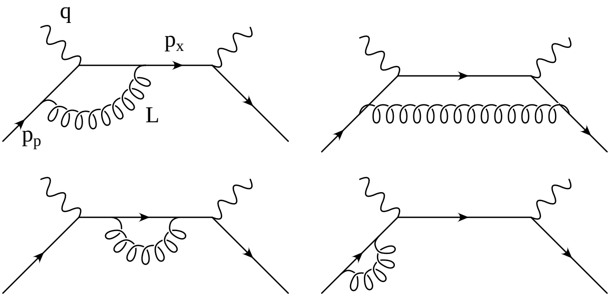
$$C^{(1)} \otimes \phi_q^{(0)} = F_{2,\mathrm{parton}}^{(1)} - C^{(0)} \otimes \phi_q^{(1)}$$

PDF:

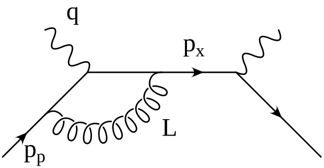
$$\phi_q^{(1)} = -\frac{1}{\epsilon}\left(P_{qq}^{(1)} \otimes \phi_q^{(0)}\right) + \phi_{q,\mathrm{bare}}^{(1)}$$

- If $C^{(1)}$ is insensitive to IR (poles), have factorization
- Can check this loop by loop

# One-loop diagrams for hadronic tensor



# Typical Loop Integral




$$I = \int [dL] \frac{1}{(L+p_x)^2} \frac{1}{(L+p_p)^2} \frac{1}{L^2}$$

- Integral depends on three scales $Q^2 \gg p_x^2 \gg p_p^2 \sim \Lambda^2_{\mathrm{QCD}}$
- Advantageous to "expand by regions" in order to factorize momentum scales at diagrammatic level

# Method of regions

1. Split loop integration into regions which aren't scaleless in dim. reg.

$$\int d^d L \to \int_{\Lambda_H} d^d L_h + \int_{\Lambda_{hc}} d^d L_{hc} + \dots$$

2. Expand under integrand in each region before evaluating
3. Integrate the expanded (one-scale) integrand over all space
4. Set scaleless integrals to zero

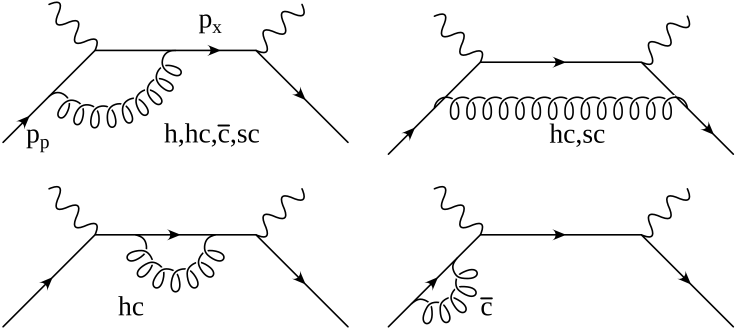
- Sum of the hard, hard-collinear, and IR regions recovers full integral, expanded for $Q^2 \gg p_x^2$
- Overlap between regions only in scaleless integrals (but must check explicitly since there's no proof)

# Regions for DIS in Breit Frame $(n_+p, p_\perp, n_-p)$

| | |
|---|---|
| **Perturbative** | $(\bar{x} \equiv 1 - x)$ |
| hard | $Q(1, 1, 1)$ |
| hard-collinear (collinear with jet) | $Q(1, \sqrt{\bar{x}}, \bar{x})$ |
| **Non-perturbative** | $(Q^2\lambda^2 \sim \Lambda^2_{\mathrm{QCD}})$ |
| anti-collinear (collinear with proton) | $Q(\lambda^2, \lambda, 1)$ |
| soft-collinear (messenger) | $Q(\lambda^2, \lambda\sqrt{\bar{x}}, \bar{x})$ |

Not everyone finds the same IR regions. Return to this later.

# Regions in individual diagrams



# Graphical representation of regions

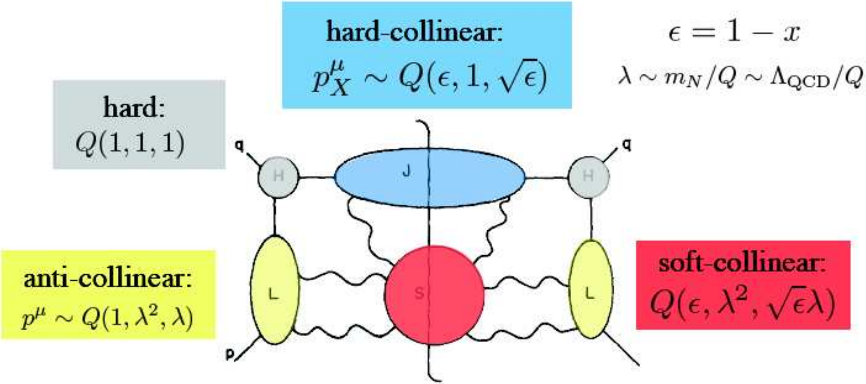



Fig. 3 1. Leading regions for DIS.

Sterman '87

Tempting to associate "soft-collinear" with "soft"

# One-loop partonic structure function (on shell)

$$W_1 = -\frac{Q^2}{p_x^2}\frac{C_F\alpha_s}{4\pi}\left[W_h + W_{hc} + W_{\bar{c}} + W_{sc}\right]$$

$$\begin{aligned}
W_h &= -\frac{4}{\epsilon^2} - \frac{6}{\epsilon} + \frac{4}{\epsilon}\ln\frac{Q^2}{\mu^2} - 2\ln^2\frac{Q^2}{\mu^2} + 6\ln\frac{Q^2}{\mu^2} + \frac{\pi^2}{3} - 16 \\
W_{hc} &= \frac{4}{\epsilon^2} + \frac{3}{\epsilon} - \frac{4}{\epsilon}\ln\frac{-p_x^2}{\mu^2} + 2\ln^2\frac{-p_x^2}{\mu^2} - 3\ln\frac{-p_x^2}{\mu^2} - \frac{\pi^2}{3} + 7 \\
W_{\bar{c}} &= \text{scaleless integral} = 0 \\
W_{sc} &= \text{scaleless integral} = 0
\end{aligned}$$

- The pole structure in the sum is $-P_{qq}/\epsilon$ at large $x$
- Poles absorbed into $\phi_q$, so have demonstrated factorization
- Finite parts go into $C$

# Factorization in SCET

# SCET $(hc, \bar{c}, sc)$

Now construct an effective theory SCET$(hc, \bar{c}, sc)$

- Introduce effective-theory fields for $hc, \bar{c}, sc$ modes
- Derive an effective-theory Lagrangian and current built out of these fields
- Treat loop integrals as in regions calculation (scaleless=0)

Diagrammatic calculations with SCET reproduce regions result

$$W_{\mathrm{SCET}(hc,\bar{c},sc)} = W_{hc} + W_{\bar{c}} + W_{sc}$$

- Equivalent to diagrammatic factorization order by order
- Abstract studies to all orders are (arguably) simpler

# Factorization procedure in SCET

1) Match hadronic tensor onto SCET$(hc, \bar{c}, sc)$

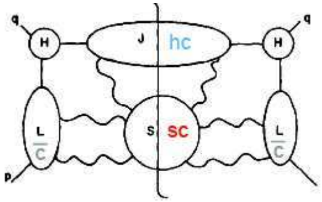
$$W^{\mu\nu}_{\mathrm{QCD}} = H(Q^2, \mu) W^{\mu\nu}_{\mathrm{SCET}(hc,\bar{c},sc)}$$

2) Use field definition to decouple *sc* from jet and calculate $J$

$$F_2 = H(Q^2, \mu) \left[ J\left(M_X^2, \mu\right) \otimes \langle N(P)|\hat{\phi}_{\mathrm{SCET}(\bar{c},sc)}|N(P)\rangle \right]$$

3) Show that the matrix element of $\hat{\phi}_{\mathrm{SCET}(\bar{c},sc)}$ is the standard PDF at large $x$

# A subtlety in matching



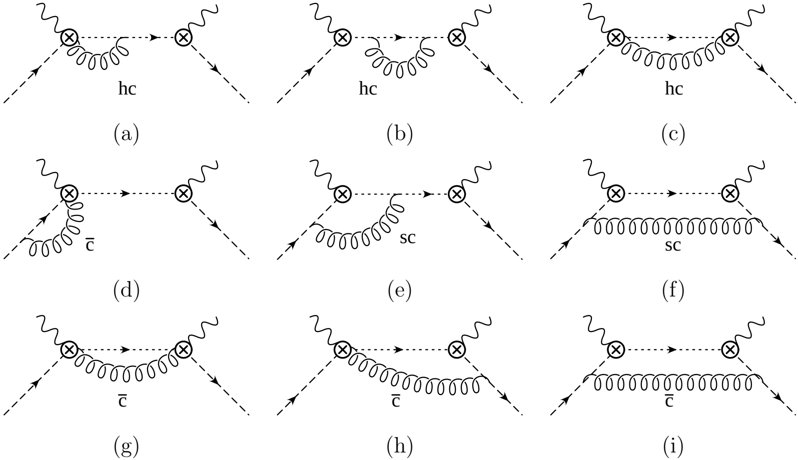

Hard fluctuations can be integrated out in the current (Manohar)

$$(\bar{\psi}\gamma^{\mu}\psi)(x) \to C_V(Q^2,\mu_h)\,\bar{\chi}_{\bar{c}}(x_-)\left[S^{\dagger}_{n_+}S_{n_-}\right](x_-)\,\gamma^{\mu}_{\perp}\chi_{hc}(x)$$

**However:** Anti-collinear gluons don't belong in the final-state jet (because $(p_{hc}+p_{\bar{c}}) \sim Q^2$)

- Anti-collinear fluctuations are virtual

# Hadronic tensor in SCET($hc, \bar{c}, sc$)




- Graphs (g-i) “forbidden” (and power suppressed)

# The hard function in SCET

Current matching

$$(\bar{\psi}\gamma^\mu\psi)(x) \to C_V(Q^2,\mu_h)\,\bar{\chi}_{\bar{c}}(x_-)\left[S^\dagger_{n_+}S_{n_-}\right](x_-)\,\gamma^\mu_\perp\chi_{hc}(x)$$

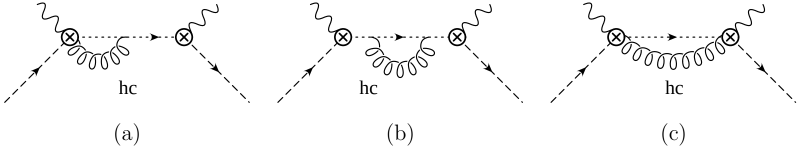
- $C_V^{(1)} \leftrightarrow W_h^{(1)}$ in regions calculation (gauge invariant)

In practice take $C_V$ from on-shell quark form factor

$$C_V(Q^2,\mu) = \lim_{\epsilon\to 0} Z_V^{-1}(\epsilon,Q^2,\mu)\,F_{\text{bare}}(\epsilon,Q^2)$$

- $C_V$ known at two loops (Becher et al; Chen, Idilbi, Ji)

# The jet function in SCET




$$J \sim \frac{1}{\pi} \mathrm{Im}\, F.T.\left[\langle 0|\, T\left\{\left(W_{hc}^{(0)\dagger}\xi_{hc}^{(0)}\right)(x)\left(\bar{\xi}_{hc}^{(0)} W_{hc}^{(0)}\right)(0)\right\}|0\rangle\right]$$

- $J^{(1)} \leftrightarrow W_{hc}^{(1)}$ in regions calculation (both gauge invariant)
- Calculated at two loops (Becher and Neubert)
- Result can be deduced from $C_V$ and $F_2$ (Chen, Idilbi, Ji)

# The PDF in SCET

Definition of $\phi_q$ as a matrix element in SCET($\bar{c}, sc$):

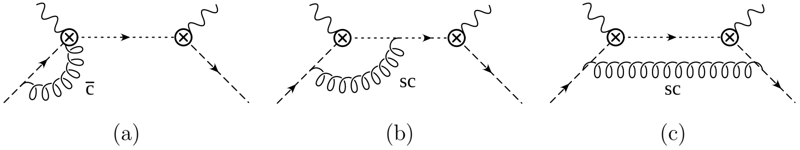
$$\phi_q(\xi,\mu)\big|_{\xi\to 1}$$
$$\frac{1}{2\pi}\int dt\, e^{-i\xi t\, n_- P}\langle N(P)|\bar{\chi}_{\bar{c}}(tn_-)[tn_-,0]_{sc}\frac{\not{n}_-}{2}\chi_{\bar{c}}(0)|N(P)\rangle$$

- The fields $\chi_{\bar{c}} = W_{\bar{c}}^{\dagger}\xi_{\bar{c}}$ communicate only through soft-collinear exchange
- Can show equivalence to QCD PDF in $\xi \to 1$ limit (Korchemsky and Marchesini '92 )

Check by determining $\mu$-dependence of $\phi_q$ perturbatively

- Find that evolution determined by $P_{qq}$ expanded for $\xi \to 1$
- Related to anomalous dimension of Wilson loop with cusps

# IR graphs in SCET($hc, \bar{c}, sc$) at one loop




- Graphs (a-c) depend on IR and must go into PDF
- Same graphs in SCET($\bar{c}, sc$) but with $hc$ propagator replaced by a soft-collinear Wilson line
- The graphs match Korchemsky's picture of PDF at large $x$ [they are $Z(p^2)\times$(Wilson loop)($\bar{x}$) ]

# PDF graphs in SCET$(hc, \bar{c}, sc)$

$$W_1^{\mathrm{IR}} = -\frac{C_F \alpha_s}{4\pi} \frac{Q^2}{p_x^2} \left[ W_{\bar{c}} + W_{sc} \right]$$

Off shell:

$$\begin{aligned} W_{\bar{c}} &= \frac{4}{\epsilon^2} + \frac{3}{\epsilon} - \frac{4}{\epsilon} \ln \frac{-p_p^2}{\mu^2} + 2 \ln^2 \frac{-p_p^2}{\mu^2} - 3 \ln \frac{-p_p^2}{\mu^2} - \frac{\pi^2}{3} + 8 \\ W_{sc} &= -\frac{4}{\epsilon^2} + \frac{4}{\epsilon} \ln \frac{p_x^2 p_p^2}{Q^2 \mu^2} - 2 \ln^2 \frac{p_x^2 p_p^2}{Q^2 \mu^2} - \pi^2 \end{aligned}$$

- Poles in $W_{\bar{c}} + W_{sc}$ independent of IR regulator $p_p^2$
- Imaginary part of sum is $P_{qq}/\epsilon$
- Factorization requires that finite part is renormalized PDF (with off-shell quarks)

# Two Questions

- How to match generic and large $x$ with SCET?
- The soft-collinear mode depends on $x$.
  Do hadronic power corrections?

# Relation between large and generic x

Two reductions at generic $\bar{x} \sim \mathcal{O}(1)$

- $p_{hc} \sim Q(1, \sqrt{\bar{x}}, \bar{x}) \sim Q(1, 1, 1) \sim p_h$
- $p_{sc} \sim Q(\lambda^2, \lambda\sqrt{\bar{x}}, \bar{x}) \sim Q(\lambda^2, \lambda, 1) \sim p_{\bar{c}}$

Methods for large-$x$ reproduce generic $x$ factorization

$$F_2 = C' \otimes \phi_q; \qquad C' \equiv [H \cdot J]$$

$C' \otimes \phi$ recovers part of generic $x$ result singular as $x \to 1$

- Factorization works the same at generic and large $x$ (no new non-perturbative information)
- Different from $B$ decay (shape function at large $x$)

# Matching large and generic $x$

**Observation:** Power corrections from hard region have no imaginary part

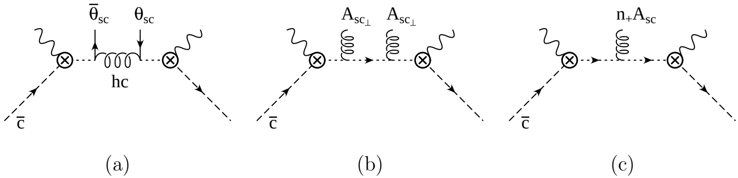
$$\frac{1}{\pi}\mathrm{Im}\, W_h \sim \frac{1}{\pi}\mathrm{Im}\,\frac{Q^2}{p_x^2}\left[W_h^{(1)} + \frac{p_x^2}{Q^2} W_h^{(1)\prime}\left(\ln\frac{Q^2}{\mu^2}\right) + \ldots\right] = H^{(1)} J^{(0)}$$

To match large and generic $x$ away from endpoint just keep more terms in $J$

- Subleading $J$ can be treated in SCET (Chay and Kim '05)
- Simpler to use method of (Chen,Idilbi, Ji '06)?

$$C^{(1)} - H^{(1)} J^{(0)} = H^{(0)}\left[J^{(1)} + \frac{p_x^2}{Q^2} J^{(1)\prime} + \ldots\right]$$

# Non-perturbative power corrections




$$\mathcal{L}^{(0)}_{hc} \sim \bar{\xi}_{hc} A^{\perp}_{hc} \partial^{\perp}_{hc} \xi_{hc} \qquad \mathcal{L}'^{(b)}_{hc+sc} \sim \bar{\xi}_{hc} A^{\perp}_{sc} \partial^{\perp}_{hc} \xi_{hc}$$

$$\frac{\mathcal{L}'^{(b)}_{hc+sc}}{\mathcal{L}_0} \sim \frac{A^{\perp}_{sc}}{A^{\perp}_{hc}} \sim \frac{\Lambda\sqrt{\bar{x}}}{Q\sqrt{\bar{x}}} \sim \lambda$$

Power corrections from soft-collinear gluons scale as $\Lambda^2/Q^2$

- Works only if “soft” IR mode is $x$-dependent (soft-collinear)
- Same $\lambda^2$ corrections are known in QCD (Ellis et al ’82)

# Summary

Discussed factorization in DIS at large $x$ with

- Purely diagrammatic analysis
- SCET formalism

Argued that:

- Factorization works much the same at large and generic $x$
- At leading order in $\Lambda/Q$ this is compatible with a "soft-collinear" IR mode
- This seems to also be true of hadronic power corrections, but only if "soft" interactions with the jet are really soft-collinear ones