SCET Applications to

Heavy-Quark Fragmentation and Drell-Yan Production

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Based on work with Thomas Becher (FNAL), Ben Pecjak (DESY), Gang Xu (Cornell) and discussions with Ignazio Scimemi (MIT)

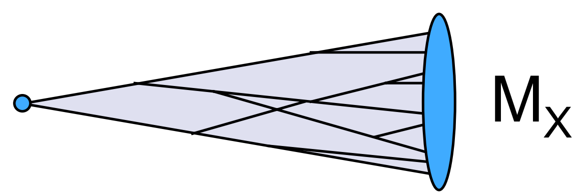
Overview

- Introduction
- Factorization in fragmentation ($x \to 1$)
- Heavy-quark fragmentation [discussions with I. Scimemi]
- Resummation of "dynamical thresholds" in Drell-Yan production [with T. Becher, G. Xu (in prep.)]
- Conclusions

All work preliminary & unpublished!

Introduction

- Generic problem in QCD:
  - Resummation for processes with >1 scales
  - Interplay of soft and collinear emissions
    $\rightarrow$ Sudakov double logarithms
  - Jet physics: $M_X^2 \ll Q^2$
    - Soft: low momentum $p^\mu \rightarrow 0$
    - Collinear: $p \parallel p_X$ with $p^2 \rightarrow 0$



  - Examples: DIS, fragmentation, Drell-Yan, Higgs production, event shapes, inclusive B decays, …

Introduction

- Will discuss two applications of momentum-space resummation technique [T. Becher, MN (2006)]
  - Much simpler than conventional approach
  - More transparent (EFT, scale separation)
  - No spurious Landau-pole singularities
  - Straightforward matching with fixed-order pQCD calculations (important)

-> see talk by T. Becher

- In these examples, new approach is an advance over existing schemes (rare in SCET…)

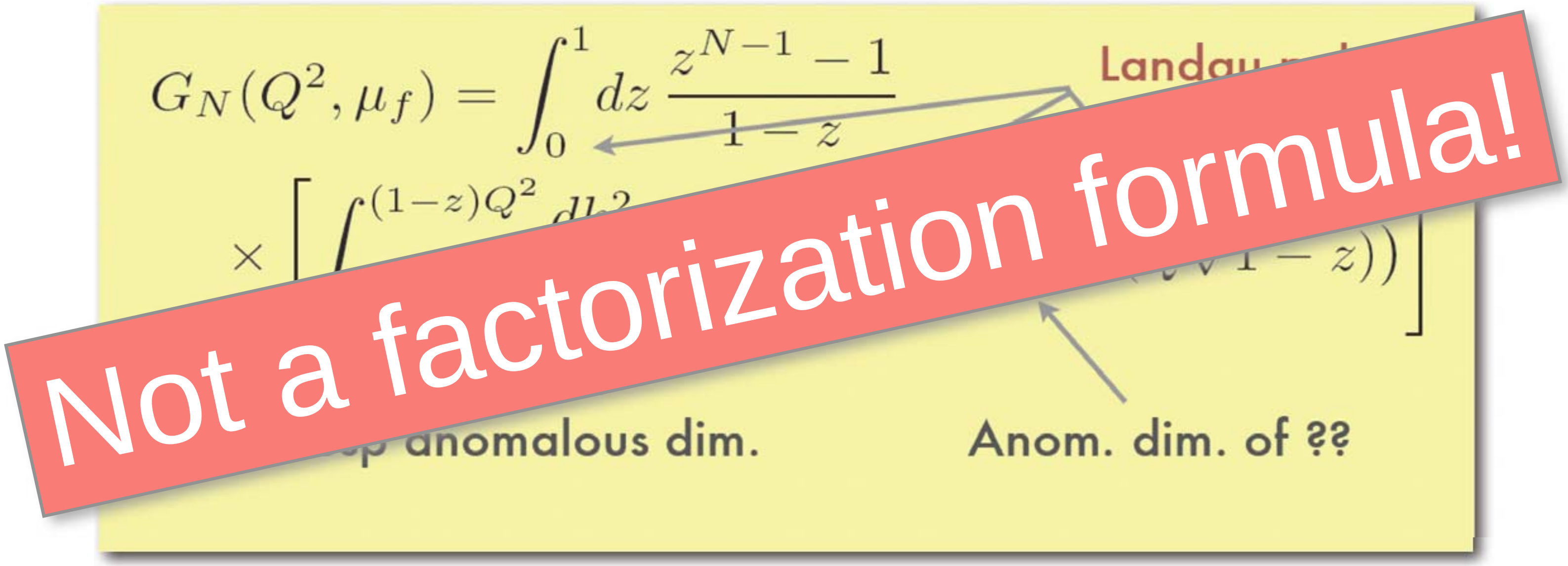
(Thomas having fun with SCET)

Introduction

- Traditionally, resummation is performed in Mellin moment space
  - Scale separation is obscure (integrals over running couplings down to zero momentum)
  - Landau poles (in Sudakov exponent and Mellin inversion)
  - Non-trivial matching with fixed-order calculations in momentum space

Introduction

- Typical resummed Sudakov exponent:



- Integrals run over Landau pole in running coupling: ambiguity ~$(\Lambda/M_X)^2$ for DIS, ~Λ/M_X for Drell-Yan

Introduction: Our approach

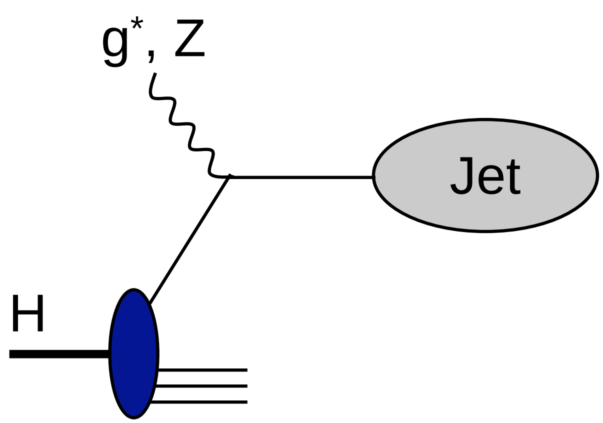
- Start from factorization formula (can be derived using SCET or other EFT)
- *Important:* objects in factorization formula defined in terms of field-theoretic objects
  $\rightarrow$ RGEs, well-defined anomalous dimensions, consistent matching, etc.
- Solve RGEs using technique based on Laplace transforms (avoid moment space!)
- Match onto fixed-order calculations
- Approach first developed for B physics, later applied to other hard QCD processes

-> see talks by T. Becher and B. Pecjak for applications to DIS

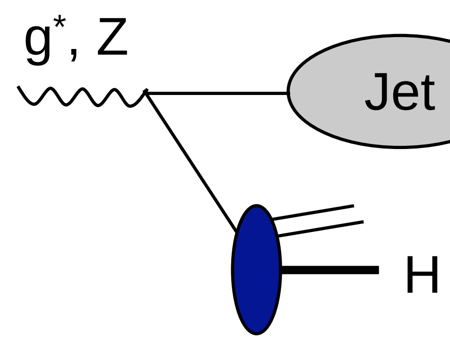
Heavy-quark fragmentation

Relation with DIS

- DIS -> see talk by B. Pecjak
- Fragmentation -> this talk



$x=-q^2/2p\cdot q$



$x=2p\cdot q/q^2$

Related by crossing, very similar kinematics, almost identical factorization formula

Factorization analysis for x→1

- Momentum regions:
  - Light-cone components $(n \cdot k, \bar{n} \cdot k, k_\perp^\mu)$ in Breit frame:

$$k^2 = n \cdot k \, \bar{n} \cdot k + k_\perp^2$$

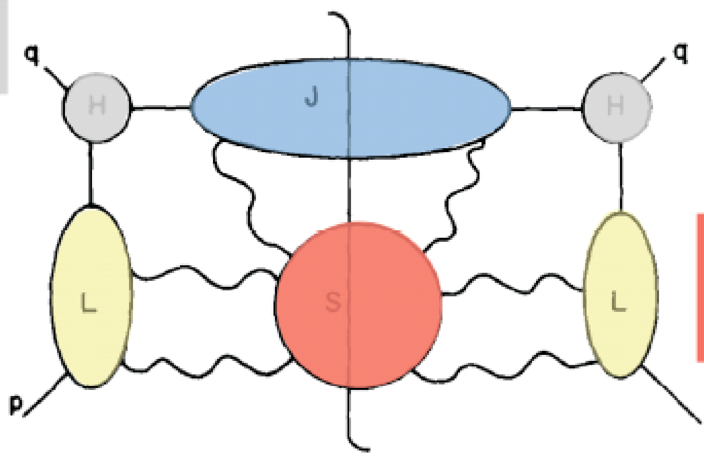
hard-collinear:
$p_X^\mu \sim Q(\epsilon, 1, \sqrt{\epsilon})$

$\epsilon = 1 - x$

$\lambda \sim m_N/Q \sim \Lambda_{QCD}/Q$

l ~ m_Q/Q for heavy quark

hard:
$Q(1,1,1)$



anti-collinear:
$p^\mu \sim Q(1, \lambda^2, \lambda)$

soft-collinear:
$Q(\epsilon, \lambda^2, \sqrt{\epsilon}\lambda)$

Factorization analysis for x→1

- QCD factorization formulae:

$$F_2(x,Q^2) = \sum_q x\, e_q^2\, |C_V(Q^2,\mu)|^2 \int_x^1 \frac{dz}{z}\, Q^2\, J(Q^2(1-z),\mu)\, \phi_{q/H}\Big(\frac{x}{z},\mu\Big) + \dots$$

$$\frac{d\sigma_H}{dx} = \sigma_{\rm Born}\, |C_V(-s,\mu)|^2 \int_x^1 \frac{dz}{z}\, s\, J(s(1-z),\mu)\, D_{q/H}\Big(\frac{x}{z},\mu\Big) + \dots$$

[Sterman (1987); Catani, Trentadue (1989); Korchemsky, Marchesini (1992); Mele, Nason (1991); Cacciari, Catani (2001)]

  - Same jet function, same hard matching coefficient (evaluated at time-like vs. space-like momentum transfer)

Factorization for x→1 in SCET

- Wilson coefficient of SCET vector current extracted from bare, on-shell QCD form factor (matching to SCET converts IR poles into UV poles): [T. Becher, MN (2006)]

$$C_V(Q^2,\mu) = \lim_{\epsilon\to 0} Z_V(\epsilon,Q^2,\mu)\, F_{\rm bare}(\epsilon,Q^2)$$

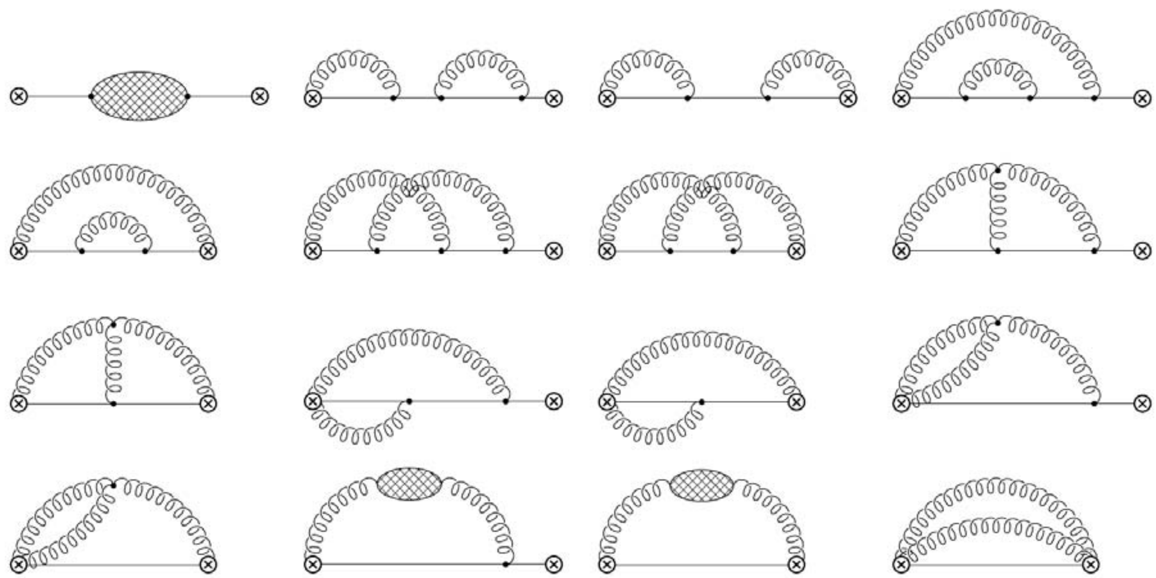
UV renormalization factor

- Result known to 2-loop order (anomalous dimension to 3 loops)

Factorization for x→1 in SCET

- SCET jet function = QCD quark propagator in light-cone gauge, known to two loops

$$\frac{\not{n}}{2}\,\bar{n}\cdot p\,\mathcal{J}(p^2) = \int d^4x\,e^{-ip\cdot x}\,\langle 0|\,\mathrm{T}\left\{\frac{\not{n}\not{\bar{n}}}{4}W^\dagger(0)\psi(0)\,\overline{\psi}(x)W(x)\frac{\not{\bar{n}}\not{n}}{4}\right\}|0\rangle$$



[T. Becher, MN (2006)]

Evolution of the hard function

- RG equation:

$$\frac{dC_V(Q^2,\mu)}{d\ln\mu} = \underbrace{\left[\Gamma_{\rm cusp}(\alpha_s)\,\ln\frac{Q^2}{\mu^2} + \gamma^V(\alpha_s)\right]}_{-2\alpha_s \frac{\partial}{\partial\alpha_s} Z_V^{(1)}(Q^2,\mu)} C_V(Q^2,\mu)$$

- Exact solution:

$$C_V(Q^2,\mu) = \exp\left[2S(\mu_h,\mu) - a_{\gamma^V}(\mu_h,\mu)\right] \times \left(\frac{Q^2}{\mu_h^2}\right)^{-a_\Gamma(\mu_h,\mu)} C_V(Q^2,\mu_h)$$

- RG functions:
  - Sudakov exponent

$$S(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha\, \frac{\Gamma_{\rm cusp}(\alpha)}{\beta(\alpha)} \int_{\alpha_s(\nu)}^{\alpha} \frac{d\alpha'}{\beta(\alpha')}$$

  - Anomalous exponent

$$a_\Gamma(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha\, \frac{\Gamma_{\rm cusp}(\alpha)}{\beta(\alpha)}$$

  - Functions of running couplings $\alpha_s(\mu)$, $\alpha_s(\nu)$

Evolution of the jet function

- Integro-differential evolution equation:

$$\frac{dJ(p^2,\mu)}{d\ln\mu} = -\left[2\Gamma_{\text{cusp}}(\alpha_s)\ln\frac{p^2}{\mu^2} + 2\gamma^J(\alpha_s)\right]J(p^2,\mu)$$

$$-\,2\Gamma_{\text{cusp}}(\alpha_s)\int_0^{p^2} dp'^2\,\frac{J(p'^2,\mu)-J(p^2,\mu)}{p^2-p'^2}$$

- Exact solution (via Laplace transformation):

$$J(p^2,\mu) = \exp\left[-4S(\mu_i,\mu)+2a_{\gamma^J}(\mu_i,\mu)\right] \times \widetilde{j}(\partial_\eta,\mu_i)\,\frac{e^{-\gamma_E\eta}}{\Gamma(\eta)}\,\frac{1}{p^2}\left(\frac{p^2}{\mu_i^2}\right)^\eta ,$$

with:

$$\eta = 2\int_\mu^{\mu_i}\frac{d\mu}{\mu}\,\Gamma_c[\alpha_s(\mu)]$$

[Becher, MN (2006)]

Resummed cross section

- In complete analogy to DIS, obtain after RG resummation:

$$\frac{d\sigma_H}{dx} = \sigma_{\text{Born}} \, |C_V(-s,\mu_h)|^2 \left(\frac{s}{\mu_h^2}\right)^{-2a_\Gamma(\mu_h,\mu_i)} \exp\left[4S(\mu_h,\mu_i) - 2a_{\gamma^V}(\mu_h,\mu_i)\right]$$
$$\times \exp\left[2a_{\gamma^\phi}(\mu_i,\mu_f)\right] \tilde{j}\left(\ln\frac{s}{\mu_i^2} + \partial_\eta, \mu_i\right) \frac{e^{-\gamma_E \eta}}{\Gamma(\eta)} \int_x^1 \frac{dz}{z} \frac{D_{Q/H}\left(\frac{x}{z},\mu_f\right)}{[(1-z)^{1-\eta}]_*}$$

-> see talk by T. Becher for definitions of RG functions

Factorization for $x \to 1$ in EFT

- Integrate out hard and hard-collinear modes in two matching steps:

Wilson coefficients:

Q^2 — $\text{QCD} \to \text{SCET}(hc,\bar{c},s)$ — $C_V(Q^2,\mu)$ (vector current matching)

$M_X^2 \approx Q^2(1-x)$ — $\text{SCET}(hc,\bar{c},s) \to \text{SCET}(\bar{c},s)$ — $J(p^2,\mu)$ (jet function)

Λ_{QCD}^2 — Low-energy matrix element = $f_{q/H}$ or $D_{q/H}$

Heavy-quark fragmentation

- Integrate out hard, hard-collinear, and heavy-quark modes in three matching steps:

Wilson coefficients:

Scale	Matching step	Wilson coefficient
Q^2	QCD → SCET(hc,$\bar{c}$,s)	$C_V(Q^2,\mu)$ (vector current matching)
$M_X^2 \approx Q^2\,(1-x)$	SCET(hc,$\bar{c}$,s) → SCET($\bar{c}$,s)	$J(p^2,\mu)$ (jet function)
m_Q^2	QCD → HQET	$C_Q(m_Q,\mu)$
$m_Q^2(1-x)^2 \sim \Lambda_{QCD}^2$	Low-energy matrix element = $S_{Q/H}$	

Similar to B®X_sg in shape-function region

[Jaffe, Randall (1994)]

Heavy-quark fragmentation: Variation I

- Integrate out hard, hard-collinear, and heavy-quark modes in three matching steps:

Wilson coefficients:

Scale	Matching step	Wilson coefficient
Q^2	QCD → SCET(hc,$\bar{c}$,s)	$C_V(Q^2,\mu)$ (vector current matching)
$M_X^2 \approx Q^2(1-x)$	SCET(hc,$\bar{c}$,s) → SCET($\bar{c}$,s)	$J(p^2,\mu)$ (jet function)
m_Q^2	QCD → HQET	$C_Q(m_Q,\mu)$
$m_Q^2(1-x)^2$		
Λ_{QCD}^2		

Hadronic effects described in terms of local HQET operators

Similar to B®X_sg in MSOPE region

Heavy-quark fragmentation: Variation II

- Integrate out hard and hard-collinear modes in two matching steps:

Wilson coefficients:

Q^2 — QCD → SCET(hc,$\bar{c}$,s) — $C_V(Q^2,\mu)$ (vector current matching)

$M_X^2 \approx Q^2(1-x) \sim m_Q^2$ — SCET(hc,$\bar{c}$,s) → HQET — $J_m(p^2,\mu)$ (new jet function)

$m_Q^2(1-x)^2 \sim \Lambda_{QCD}^2$ — Low-energy matrix element = $S_{Q/H}$

- New jet function

Factorization of the fragmentation function

I. Decoupling of $Q\bar{Q}$ pairs

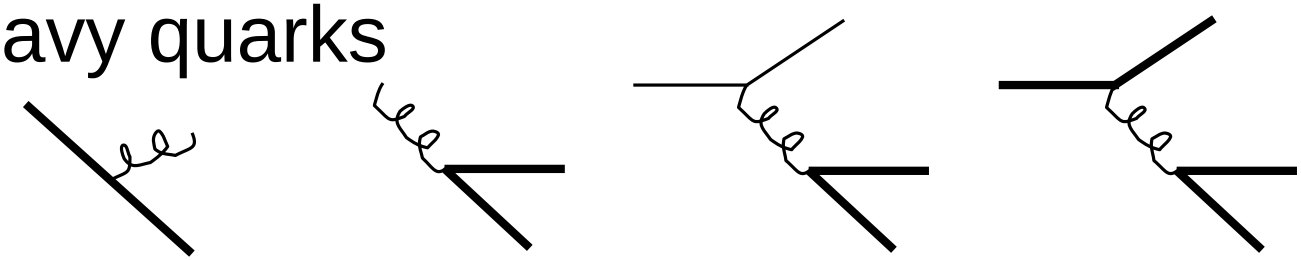
- Generic form of differential cross section for e^+e^-®V®H+X is (with x=2E_H/Ös):

$$\frac{d\sigma_H}{dx} = \sum_a \int_x^1 \frac{dz}{z} \frac{d\hat{\sigma}_a}{dz}(z, \sqrt{s}, \mu)\, D_{a/H}\left(\frac{x}{z}, m_Q, \mu\right)$$

Cross section for producing massless parton a with energy fraction z

Probability for massless parton a to fragment into heavy hadron H

- Possibilities: a=Q, g, q, $\bar{q}$, $\bar{Q}$
- All except a=Q involve pair production of heavy quarks



I. Decoupling of $Q\bar{Q}$ pairs

- Integrate out heavy-quark pairs (real and virtual) by matching (n_l+1)-flavor QCD onto n_l-flavor QCD:

$$D_{a/H}^{(n_l+1)}(x, m_Q, \mu) = C_{a/Q}(x, m_Q, \mu) \otimes D_{Q/H}^{(n_l)}(x, m_Q, \mu)$$

Conversion of parton a into heavy quark Q

Single fragmentation function in "quenched QCD"

along with:

$$\alpha_s^{(n_l+1)}(\mu) = \alpha_s^{(n_l)}(\mu) \left[1 + \frac{2}{3} T_F \frac{\alpha_s^{(n_l)}(\mu)}{2\pi} \ln \frac{\mu^2}{m_Q^2} + \dots \right]$$

I. Decoupling of $Q\bar{Q}$ pairs

- Matching reveals interesting subtlety:

$$C_{Q/Q}(x, m_Q, \mu) = \delta(1-x) + \left(\frac{\alpha_s^{(n_l)}(\mu)}{2\pi}\right)^2 \left[C_F T_F\, c_2(x) + \text{non-singular terms}\right]$$

$$C_{g/Q}(x, m_Q, \mu) = \frac{\alpha_s^{(n_l)}(\mu)}{2\pi} T_F \left[x^2 + (1-x)^2\right] \ln\frac{\mu^2}{m_Q^2} + O(\alpha_s^2)\,,$$

$$C_{a/Q}(x, m_Q, \mu) = O(\alpha_s^2)\,; \quad a = \bar{Q}, q, \bar{q}.$$

[obtained using: Melnikov, Mitov (2004)]

with:

$$c_2(x) = \delta(1-x)\left[\frac{3139}{648} - \frac{\pi^2}{3} + \frac{2}{3}\zeta_3 + \frac{1}{2}\ln^2\frac{\mu^2}{m_Q^2} - \left(\frac{1}{6} + \frac{2\pi^2}{9}\right)\ln\frac{\mu^2}{m_Q^2}\right]$$
$$+ \frac{1}{(1-x)_+}\left[\frac{56}{27} + \frac{2}{3}\ln^2\frac{\mu^2}{m_Q^2} - \frac{20}{9}\ln\frac{\mu^2}{m_Q^2}\right].$$

Generates large log in matching condition!

I. Decoupling of $Q\bar{Q}$ pairs

- Soft dynamics in heavy hadron implies that soft fragmentation function peaks at

$$1\text{-}x_{peak} \sim \Lambda_{QCD}/m_Q$$

- Appears to implies that $D_{Q/H}^{(n_l+1)}(x,m)$ contains IR terms $\ln(1\text{-}x) \sim \ln(\Lambda_{QCD}/m_Q)$ not present in $D_{Q/H}^{(n_l)}(x,m)$

?!?

I. Decoupling of $Q\bar{Q}$ pairs

- Wrong interpretation!
- Large logs have UV origin and are needed to convert $a_s(m)$ in expression for hard cross section from (n_l+1)-flavor to n_l-flavor theory
- Define:

$$\frac{d\hat{\sigma}_Q^{(n_l)}}{dx}(x,\sqrt{s},m_Q,\mu) \equiv \sum_a \frac{d\hat{\sigma}_a^{(n_l+1)}}{dx}(x,\sqrt{s},\mu) \otimes C_{a/Q}(x,m_Q,\mu)$$

$$\frac{d\sigma_H}{dx} = \frac{d\hat{\sigma}_Q^{(n_l)}}{dx}(x,\sqrt{s},m_Q,\mu) \otimes D_{Q/H}^{(n_l)}(x,m_Q,\mu)$$

Fragmentation function in “quenched QCD”

II. Matching onto HQET

- Fragmentation function in “quenched QCD” can be matched (locally!) onto HQET:

$$D_{Q/H}(x, m_Q, \mu)\,\frac{dx}{x} = C_D(m_Q, \mu)\,S_{Q/H}(\omega, \mu)\,d\omega + \text{power corrections}$$

where: $$\omega = M_H\left(\frac{1}{x} - \frac{m_Q}{M_H}\right)\,, \qquad x = \frac{M_H}{m_Q + \omega}$$

  - Matching coefficient can be extracted by computing “partonic” expressions for the fragmentation functions in the two theories

II. Matching onto HQET

- Definitions:

$$D_{Q/H}(x, m_Q, \mu) = \frac{x^{d-3}}{2\pi} \int dt\, e^{ip\cdot nt} \sum_{\chi} \frac{\not{n}_{\alpha\beta}}{2} \langle 0|(U_n^\dagger \psi_Q)^i_\beta(tn)|H(p_H)\chi\rangle\langle H(p_H)\chi|(\bar{\psi}_Q U_n)^i_\alpha(0)|0\rangle$$

[Collins, Soper (1982)]

$$S_{Q/H}(\omega, \mu) = \frac{n\cdot v}{2\pi} \int dt\, e^{ik\cdot nt} \sum_{\chi_s} \langle 0|(S_n^\dagger h_v)^i_\alpha(tn)|H_\infty(v)\chi_s\rangle\langle H_\infty(v)\chi_s|(\bar{h}_v S_n)^i_\alpha(0)|0\rangle$$

[Jaffe, Randall (1994)]

- Can be related to (restricted) discontinuities of two-point functions

II. Matching onto HQET

- Wilson loop analysis shows that "partonic" expression for HQET fragmentation function $S_{Q/H}$(w,m) coincides with "partonic" expression for shape function S(-w,m) to all orders in perturbation theory:

$$S^{\rm PT}(\omega,\mu) = \frac{n\cdot v}{2\pi}\int dt\, e^{in\cdot v\omega t}\,\frac{1}{N_c}\,\langle\bar{0}|\,{\rm Tr}\,(S_v^\dagger S_n)(0)(S_n^\dagger S_v)(tn)|0\rangle$$

$$S^{\rm PT}_{Q/H}(\omega,\mu) = \frac{n\cdot v}{2\pi}\int dt\, e^{in\cdot v\omega t}\,\frac{1}{N_c}\,\langle 0|\,{\rm Tr}\,(S_n^\dagger S_v)(tn)(S_v^\dagger S_n)(0)|0\rangle$$

[also noted in: Gardi (2005)]

II. Matching onto HQET

- Two-loop expression for $D_{Q/H}^{PT}(x,m)$ known

  [Melnikov, Mitov (2004)]

- Two-loop expression for $S_{Q/H}^{PT}(w,m)$ obtained from two-loop "partonic" shape function

  [T. Becher, MN (2005)]

- Resulting Wilson coefficient, with $L=\ln(m/m_Q)$:

$$C_D(m_Q,\mu) = 1 + \frac{C_F\alpha_s(\mu)}{\pi}\left(L^2 + \frac{L}{2} + 1 + \frac{\pi^2}{24}\right)$$

$$+ C_F\left(\frac{\alpha_s(\mu)}{\pi}\right)^2\left[C_F H_F + C_A H_A + T_F n_f H_f\right]$$

II. Matching onto HQET

- With:

$$H_F = \frac{L^4}{2} + \frac{L^3}{2} + \left(\frac{9}{8} + \frac{\pi^2}{24}\right) L^2 + \left(\frac{11}{16} - \frac{11\pi^2}{48} + 3\zeta_3\right) L$$

$$+ \frac{241}{128} + \left(\frac{13}{48} - \frac{\ln 2}{2}\right)\pi^2 - \frac{163\pi^4}{5760} - \frac{3}{8}\zeta_3$$

$$H_A = \frac{11L^3}{18} + \left(\frac{167}{72} - \frac{\pi^2}{12}\right) L^2 + \left(\frac{1165}{432} + \frac{7\pi^2}{18} - \frac{15}{4}\zeta_3\right) L$$

$$+ \frac{12877}{10368} + \left(\frac{755}{1728} + \frac{\ln 2}{4}\right)\pi^2 - \frac{47\pi^4}{2880} + \frac{89}{144}\zeta_3$$

$$H_f = -\frac{2L^3}{9} - \frac{13L^2}{18} - \left(\frac{77}{108} + \frac{\pi^2}{9}\right) L - \frac{1541}{2592} - \frac{37\pi^2}{432} - \frac{13}{36}\zeta_3$$

III. Evolution and resummation

- HQET fragmentation function obeys same evolution equation as shape function
- Exact solution: [S. Bosch, B. Lange, MN, G. Paz et al. (2004)]

$$C_D(m_Q,\mu) = \exp\left[-2S(\mu_h,\mu) - 2a_{\gamma^\phi+\gamma^s}(\mu_h,\mu)\right]\left(\frac{m_Q}{\mu_h}\right)^{2a_\Gamma(\mu_h,\mu)} C_D(m_Q,\mu_h)$$

$$S_{Q/H}(\hat\omega,\mu) = \exp\left[2S(\mu_0,\mu) + 2a_{\gamma^s}(\mu_0,\mu)\right]\frac{e^{-\gamma_E\eta}}{\Gamma(\eta)}\int_0^{\hat\omega} d\hat\omega'\,\frac{S_{Q/H}(\hat\omega',\mu_0)}{\mu_0^\eta(\hat\omega-\hat\omega')^{1-\eta}}\,,$$

III. Evolution and resummation

- Using these results, one obtains an explicit, exact RG-improved expression for the differential e^+e^-®H+X cross section (for x®1) directly in momentum space
- As always in our scheme, matching onto fixed-order expressions valid away from the endpoint is straightforward
- Numerical results to follow …

Resummation of dynamical thresholds in Drell-Yan production

[with T. Becher, G. Xu (in prep.)]

Differential cross section

- Study N_1+N_2®g*®l^+l^-+X and measure invariant mass M and rapidity Y of lepton pair
- pQCD formula (known at NNLO):

$$\frac{d^2\sigma}{dM^2dY} = \frac{4\pi\alpha^2}{3N_cM^2s}\sum_{i,j}\int dydz\,\frac{C_{ij}(y,z,M,\mu)}{[1-y(1-z)][1-(1-y)(1-z)]}\,f_i(x_1,\mu)\,f_j(x_2,\mu)$$

with: t=M^2/s , z=$M^2/\hat{s}$=t/(x_1x_2)Î[0,1]

$$x_1 = \sqrt{\frac{\tau}{z}\,\frac{1-(1-y)(1-z)}{1-y(1-z)}}\,e^Y\,, \qquad x_2 = \sqrt{\frac{\tau}{z}\,\frac{1-y(1-z)}{1-(1-y)(1-z)}}\,e^{-Y}$$

[Anastasiou, Dixon, Melnikov, Petriello (2003)]

Differential cross section

- NLO kernels:

$$\frac{C_{q\bar{q}}}{e_q^2} = \delta(1-z)\,\frac{\delta(y)+\delta(1-y)}{2}\left[1+\frac{C_F\alpha_s}{\pi}\left(\frac{3}{2}\ln\frac{M^2}{\mu^2}+\frac{\pi^2}{3}-4\right)\right]$$

$$+\frac{C_F\alpha_s}{2\pi}\left\{[\delta(y)+\delta(1-y)]\left[4\left(\frac{\ln(1-z)}{1-z}\right)_+ + \frac{1+z^2}{(1-z)_+}\ln\frac{M^2}{\mu^2}\right.\right.$$

$$\left. -2(1+z)\ln(1-z) - \frac{1+z^2}{1-z}\ln z + 1 - z\right]$$

$$\left. + \left[1+\frac{(1-z)^2}{z}y(1-y)\right]\left[\frac{1+z^2}{1-z}\left(\frac{1}{(y)_+}+\frac{1}{(1-y)_+}\right)-2(1-z)\right]\right\}$$

$$\frac{C_{qg}}{e_q^2} = \frac{T_F\alpha_s}{2\pi}\left\{\delta(y)\left[\left(z^2+(1-z)^2\right)\left(\ln\frac{M^2}{\mu^2}+\ln\frac{(1-z)^2}{z}\right)+2z(1-z)\right]\right.$$

$$\left. + \left[1+\frac{(1-z)^2}{z}y(1-y)\right]\left[\left(z^2+(1-z)^2\right)\frac{1}{(y)_+}+2z(1-z)+(1-z)^2 y\right]\right\}$$

True threshold region

- Threshold region:

  $t=M^2/s$®1, which implies: $z=t/(x_1x_2)$®1

- Factorization formula (derivable in SCET):

$$\frac{d\sigma}{dM^2} = \frac{4\pi\alpha^2}{3N_c M^2 s} |C_V(-M^2,\mu)|^2 \sum_q e_q^2 \int \frac{dx_1}{x_1}\frac{dx_2}{x_2} \left[f_q(x_1,\mu)\, f_{\bar{q}}(x_2,\mu) + (q \leftrightarrow \bar{q}) \right]$$
$$\times \sqrt{\hat{s}}\, W_{\rm DY}\big(\sqrt{\hat{s}}(1-z),\mu\big)\,,$$

Time-like Wilson loop
(analog of DIS jet function)

  with $x_1x_2$³t

- Generalization to rapidity distribution easy, since $Y=1/2\ln(x_1/x_2)$ at leading power in $(1-z)$

True threshold region

- Resummation in moment space: [Bolzoni, hep-ph/0609073]

$$\sigma(N,Q^2,M) \equiv \int_0^1 dx\, x^{N-1} \int_{\log\sqrt{x}}^{\log 1/\sqrt{x}} dY\, e^{iMY} \sigma(x,Q^2,Y)$$
$$= F_1^{H_1}(N+iM/2,\mu^2) F_2^{H_2}(N-iM/2,\mu^2) C\left(N,\frac{Q^2}{\mu^2},\alpha_s(\mu^2),M\right)$$

with (complex!) moments:

$$F_i^{H_i}(N \pm iM/2,\mu^2) = \int_0^1 dx\, x^{N-1\pm iM/2} F_i^{H_i}(x,\mu^2),$$
$$C\left(N,\frac{Q^2}{\mu^2},\alpha_s(\mu^2),M\right) = 2\int_0^1 dz\, z^{N-1} \int_0^{\log 1/\sqrt{z}} dy \cos(My)\, C\left(z,\frac{Q^2}{\mu^2},\alpha_s(\mu^2),y\right)$$

Mellin inversion obviously challenging!

True threshold region

- Traditional resummation, using $y_{\text{(parton)}}=0$ to leading power:

$$C_I^{res}\left(N,\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right)=\exp\left\{-\int_1^{N^2}\frac{dn}{n}\left[\int_{n\mu^2}^{Q^2}\frac{dk^2}{k^2}\left(A_1\alpha_s(\frac{k^2}{n})+A_2\alpha_s^2(\frac{k^2}{n})\right)+B_1\alpha_s(\frac{Q^2}{n})\right]\right\}$$

exhibits usual problems

However, all this is meaningless!

"Dynamical thresholds"

- In practice, we are not interested in the region where M^2/s®1 !
- Has been argued that, for steeply fallings PDFs, cross section receives dominant contributions from region where z®1, even though $x_{1,2}$ not near 1 (even very small) [Catani et al.]
- In fact, we find that leading terms for z®1 reproduce about 95% of NLO correction for Ös=40 GeV and M=8 GeV

"Dynamical thresholds"

- For $x_{1,2}$ not close to 1, distribution of parton variables is sensitive to subleading terms in (1-z):

$$x_1 = \sqrt{\frac{\tau}{z}\,\frac{1-(1-y)(1-z)}{1-y(1-z)}}\,e^{Y}\,, \qquad x_2 = \sqrt{\frac{\tau}{z}\,\frac{1-y(1-z)}{1-(1-y)(1-z)}}\,e^{-Y}$$

- Deviations from Born values are O(1-z), but needed to provide dynamical suppression!

"Blind resummation"

- People have ignored these subtleties and "blindly" applied moment-space results away from true threshold region
- Matching to fixed-order result done numerically:

$$\frac{d\sigma}{dQ^2dY} = \frac{d\sigma^{FO}}{dQ^2dY} + \frac{d\sigma^{res}}{dQ^2dY} - \left[\frac{d\sigma^{res}}{dQ^2dY}\right]_{\alpha_s=0} - \alpha_s\left[\frac{\partial}{\partial\alpha_s}\left(\frac{d\sigma^{res}}{dQ^2dY}\right)\right]_{\alpha_s=0}$$

- Fact that N®¥ forces $x_{1,2}$®1 was ignored

“Blind resummation”

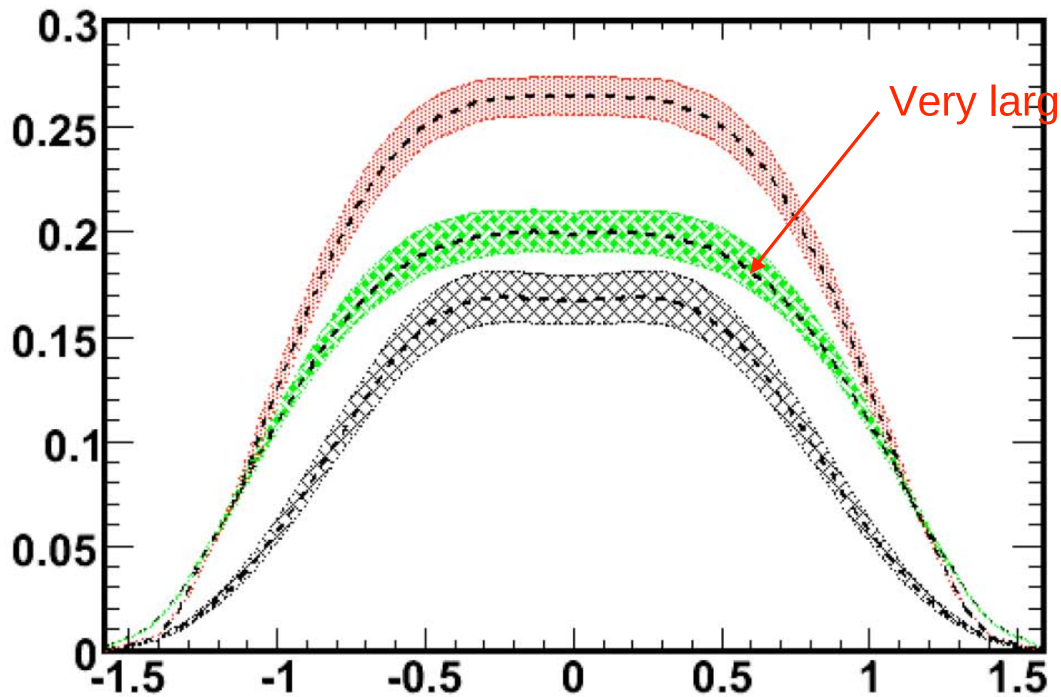


Figure 2: Y-dependence of $d^2\sigma/(dQ^2dY)$ in units of pb/GeV^2. The curves are, from top to bottom, the NLO result (red band), the NLO+NLL resummation (green band) and the LO (black band). The bands are obtaine as in figure 1.

[Bolzoni, hep-ph/0609073]

Careful matching with pQCD

- EFT factorization consistent with pQCD results:

$$\frac{d^2\sigma}{dM^2dY} = \frac{4\pi\alpha^2}{3N_cM^2s}|C_V(-M^2,\mu)|^2\sum_q e_q^2\int\frac{dz}{z}\sqrt{\hat{s}}\,W_{\rm DY}\big(\sqrt{\hat{s}}(1-z),\mu\big)$$
$$\times\left[\frac{f_q(\sqrt{\tau}\,e^Y,\mu)\,f_{\bar{q}}(\sqrt{\tau}/z\,e^{-Y},\mu)+f_q(\sqrt{\tau}/z\,e^Y,\mu)\,f_{\bar{q}}(\sqrt{\tau}\,e^{-Y},\mu)}{2}+(q\leftrightarrow\bar{q})\right]$$

- Capability to match analytically with fixed-order result crucial!

Resummation in EFT

- Applying momentum-space resummation technique, we get:

$$
\begin{aligned}
&|C_V(-M^2,\mu)|^2\,\sqrt{\hat{s}}\,W_{\rm DY}\big(\sqrt{\hat{s}}(1-z),\mu\big)\\
&\to e^{4S(\mu_h,\mu_i)-2a_{\gamma^V}(\mu_h,\mu_i)}\left(\frac{M^2}{\mu_h^2}\right)^{-2a_\Gamma(\mu_h,\mu_i)}|C_V(-M^2,\mu_h)|^2\\
&\times\; e^{4a_{\gamma\phi}(\mu_i,\mu)}\,\frac{z^{-\eta}}{(1-z)^{1-2\eta}}\,\widetilde{j}_{\rm DY}\left(\ln\frac{M^2(1-z)^2}{\mu_i^2 z}+\partial_\eta,\mu_i\right)\frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)}
\end{aligned}
$$

-> see talk by T. Becher for definitions of RG functions

Numerical results to follow …

Conclusions

- Methods from effective field theory provide powerful, efficient tools to study factorization and resummation in many hard QCD processes
- Have resummed Sudakov logarithms directly in momentum space by solving RGEs in EFT
- Results agree with traditional approach at every fixed order in perturbation theory, but are free of spurious Landau-pole singularities
- Easier to match with FOPT results for differential cross sections away from threshold region