Reparameterization Invariant Operators in SCET

Ongoing work of
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OUTLINE

- Reparameterization invariant in SCET
- Construction of RPI Operators
- Constraints in light-light currents

REPARAMETRIZATION INVARIANT IN SCET

The total P^μ momentum of collinear particle is decompose into a sum of a large momentum p^μ and a ultrasoft momentum k^μ

$$P^\mu \; = \; p^\mu + k^\mu = \frac{n^\mu}{2}\bar{n}\cdot(p+k) + \frac{\bar{n}^\mu}{2} n\cdot k + (p_\perp + k_\perp)$$

$$n^2 = 0 \quad \bar{n}^2 = 0 \quad n\cdot\bar{n} = 2 \qquad \bar{n}\cdot p \sim \lambda^0,\, p_\perp^\mu \sim \lambda,\, n\cdot p = 0 \qquad k^\mu \sim \lambda^2$$

Two types of ambiguities:

(a) $\bar{n}\cdot(p+k)$ and $(p_\perp^\mu + k_\perp^\mu)$ are arbitrary by an order $Q\lambda^2$ amount.

$$i\bar{n}\cdot D_c \to i\bar{n}\cdot D_c + W i\bar{n}\cdot D_{us} W^\dagger$$
$$iD_c^\perp \to iD_c^\perp + W iD_{us}^\perp W^\dagger$$

✓ (b)Any choice of n and $\bar{n}$ satisfying $n^2 = 0$, $\bar{n}^2 = 0$, $n\cdot\bar{n} = 2$ are equally good.

Chay, Kim hep-ph/0201197

Manohar, Mehen, Pirjol, Stewart hep-ph/0204229

The most general infinitesimal change in n and $\bar{n}$ which preserves

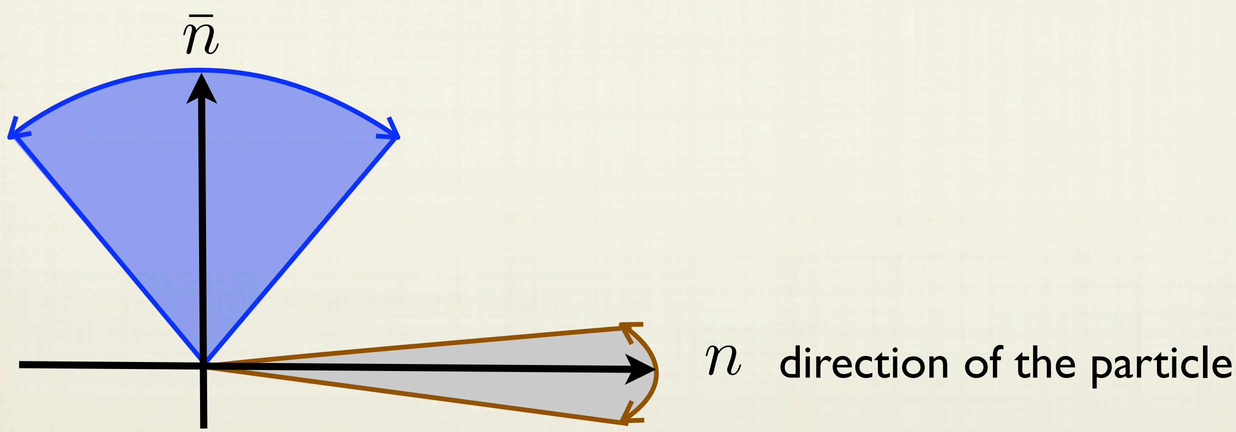
$n^2 = 0 \quad \bar{n}^2 = 0 \quad n \cdot \bar{n} = 2$ is

$$\text{(I)} \begin{cases} n_\mu \to n_\mu + \Delta_\mu^\perp \\ \bar{n}_\mu \to \bar{n}_\mu \end{cases} \qquad \text{(II)} \begin{cases} n_\mu \to n_\mu \\ \bar{n}_\mu \to \bar{n}_\mu + \varepsilon_\mu^\perp \end{cases} \qquad \text{(III)} \begin{cases} n_\mu \to (1+\alpha)\, n_\mu \\ \bar{n}_\mu \to (1-\alpha)\, \bar{n}_\mu \end{cases}$$

$$\bar{n}\cdot\varepsilon^\perp = n\cdot\varepsilon^\perp = \bar{n}\cdot\Delta^\perp = n\cdot\Delta^\perp = 0$$

To ensure the correct power counting we take $\{\Delta^\perp, \varepsilon^\perp, \alpha\} \sim \{\lambda, \lambda^0, \lambda^0\}$

This implies that collinear particles remain collinear under RP



TRANSFORMATIONS UNDER RPI

Type (I)	Type (II)	Type (III)
$n \to n + \Delta^\perp$	$n \to n$	$n \to n + \alpha\, n$
$\bar{n} \to \bar{n}$	$\bar{n} \to \bar{n} + \varepsilon^\perp$	$\bar{n} \to \bar{n} - \alpha\, \bar{n}$
$n \cdot D \to n \cdot D + \Delta^\perp \cdot D^\perp$	$n \cdot D \to n \cdot D$	$n \cdot D \to n \cdot D + \alpha\, n \cdot D$
$D_\mu^\perp \to D_\mu^\perp - \frac{\Delta_\mu^\perp}{2}\, \bar{n} \cdot D - \frac{\bar{n}_\mu}{2} \Delta^\perp \cdot D^\perp$	$D_\mu^\perp \to D_\mu^\perp - \frac{\varepsilon_\mu^\perp}{2}\, n \cdot D - \frac{n_\mu}{2}\, \varepsilon^\perp \cdot D^\perp$	$D_\mu^\perp \to D_\mu^\perp$
$\bar{n} \cdot D \to \bar{n} \cdot D$	$\bar{n} \cdot D \to \bar{n} \cdot D + \varepsilon^\perp \cdot D^\perp$	$\bar{n} \cdot D \to \bar{n} \cdot D - \alpha\, \bar{n} \cdot D^\perp$
$\xi_n \to \left(1 + \frac{1}{4}\, \not{\Delta}^\perp \not{\bar{n}}\right) \xi_n$	$\xi_n \to \left(1 + \frac{1}{2} \not{\varepsilon}^\perp \frac{1}{\bar{n} \cdot D}\, \not{D}^\perp\right) \xi_n$	$\xi_n \to \xi_n$
$W \to W$	$W \to \left[\left(1 - \frac{1}{\bar{n} \cdot D}\, \epsilon^\perp \cdot D^\perp\right) W\right]$	$W \to W$

- The vector itself remains invariant $D^\mu \longrightarrow D^\mu$
- The quark field $\psi(x) = \sum_p e^{-ip\cdot x} \left[1 + \frac{1}{\bar{n} \cdot D} \not{D}^\perp \frac{\not{\bar{n}}}{2}\right] \xi_{n,p}$ remains invariant

WHAT IS RPI USEFUL FOR? Connect operators in a OPE

An example: scalar chiral-even operator $S(q)$

Expansion in SCET $S(q) = C\mathcal{J}_V + \sum_{i=1}^{2} D_i\,(q_\alpha \mathcal{V}_i^\alpha) + \sum_{i=1}^{2} \tilde{D}_i\,(q_\alpha \tilde{\mathcal{V}}_i^\alpha) + E\,(q_\alpha \mathcal{V}_3^\alpha)$

$$\mathcal{J}_V(\vec{\omega}) = \bar{\chi}_{n,\omega_1} \frac{\bar{\not{n}}}{2} \chi_{n,\omega_2} \quad \text{LO}$$

Hardmeier, Lunghi, Pirjol, Wyler hep-ph/0307171

$$\left.\begin{aligned}
\mathcal{V}_1^\alpha(\vec{\omega}) &= \left[\bar{\xi}_n \frac{\bar{\not{n}}}{2} (i\not{D}_{\perp c})^\dagger W_n\right]_{\omega_1} \frac{1}{\bar{n}\cdot\mathcal{P}^\dagger} \gamma^\alpha \chi_{n,\omega_2} + \bar{\chi}_{n,\omega_1} \gamma^\alpha \frac{1}{\bar{n}\cdot\mathcal{P}} \left[W_n^\dagger i\not{D}_{\perp c} \frac{\bar{\not{n}}}{2} \xi_n\right]_{\omega_2} \\
\mathcal{V}_2^\alpha(\vec{\omega}) &= \left[\bar{\xi}_n \frac{\bar{\not{n}}}{2} (iD_{\perp c}^\alpha)^\dagger W_n\right]_{\omega_1} \frac{1}{\bar{n}\cdot\mathcal{P}^\dagger} \chi_{n,\omega_2} + \bar{\chi}_{n,\omega_1} \frac{1}{\bar{n}\cdot\mathcal{P}} \left[W_n^\dagger iD_{\perp c}^\alpha \frac{\bar{\not{n}}}{2} \xi_n\right]_{\omega_2} \\
\mathcal{V}_3^\alpha(\vec{\omega}) &= \bar{\chi}_{n,\omega_1} \frac{\bar{\not{n}}}{2} \left[\frac{1}{\bar{n}\cdot\mathcal{P}} W^\dagger iD_\perp^\alpha W\right]_{\omega_3} \chi_{n,\omega_2}
\end{aligned}\right\} \quad \text{NLO}$$

We impose RPI: $\delta_{RP} S(q) = 0$

$$\downarrow$$

$$\delta_{RP}\left[C\mathcal{J}_V + \sum_{i=1}^{2} D_i\,(q_\alpha \mathcal{V}_i^\alpha) + \sum_{i=1}^{2} \tilde{D}_i\,(q_\alpha \tilde{\mathcal{V}}_i^\alpha) + E\,(q_\alpha \mathcal{V}_3^\alpha)\right] = 0$$

All the Wilson coefficients are connected!

$$J^{(0)}(\omega) = \bar{\chi}_{n,\omega}\Gamma\mathcal{H}_v\,,$$

$$J^{(1a)}(\omega) = \frac{1}{\omega}\,\bar{\chi}_{n,\omega}\mathcal{P}_\alpha^{\perp\dagger}\Theta^\alpha_{(a)}\mathcal{H}_v$$

$$J^{(1b)}(\omega_{1,2}) = \frac{1}{m}\,\bar{\chi}_{n,\omega_1}(ig\mathcal{B}^\perp_\alpha)_{\omega_2}\Theta^\alpha_{(b)}\mathcal{H}_v\,.$$

$$J^{(2a)}(\omega) = \frac{1}{2m}\,\bar{\chi}_{n,\omega}\Upsilon^\sigma_{(a)} i\mathcal{D}^T_{us\,\sigma}\mathcal{H}_v\,,$$

$$J^{(2b)}(\omega) = -\frac{n\cdot v}{\omega}\,\bar{\chi}_{n,\omega}\, i\bar{n}\cdot\overleftarrow{\mathcal{D}}_{us}\Upsilon_{(b)}\mathcal{H}_v\,,$$

$$J^{(2c)}(\omega) = -\frac{1}{\omega}\,\bar{\chi}_{n,\omega} i\overleftarrow{\mathcal{D}}^\perp_{us\,\alpha}\,\Upsilon^\alpha_{(c)}\mathcal{H}_v\,,$$

$$J^{(2d)}(\omega) = \frac{1}{\omega^2}\,\bar{\chi}_{n,\omega}\mathcal{P}_\alpha^{\perp\dagger}\mathcal{P}_\beta^{\perp\dagger}\Upsilon^{\alpha\beta}_{(d)}\mathcal{H}_v\,,$$

$$J^{(2e)}(\omega_{1,2}) = \frac{1}{m(\omega_1+\omega_2)}\,\bar{\chi}_{n,\omega_1}(ig\mathcal{B}^\perp_\alpha)_{\omega_2}\mathcal{P}_\beta^{\perp\dagger}\Upsilon^{\alpha\beta}_{(e)}\mathcal{H}_v\,,$$

$$J^{(2f)}(\omega_{1,2}) = \frac{\omega_2}{m(\omega_1+\omega_2)}\,\bar{\chi}_{n,\omega_1}\left(\frac{\mathcal{P}^\perp_\beta}{\omega_2}+\frac{\mathcal{P}^{\perp\dagger}_\beta}{\omega_1}\right)(ig\mathcal{B}^\perp_\alpha)_{\omega_2}\Upsilon^{\alpha\beta}_{(f)}\mathcal{H}_v\,,$$

$$J^{(2g)}(\omega_{1,2}) = \frac{1}{m\,n\cdot v}\,\bar{\chi}_{n,\omega_1}\left\{(ign\cdot\mathcal{B})_{\omega_2}+2(ig\mathcal{B}_\perp)_{\omega_2}\cdot\mathcal{P}^\dagger_\perp\frac{1}{\bar{\mathcal{P}}^\dagger}\right\}\Upsilon_{(g)}\mathcal{H}_v\,,$$

$$J^{(2h)}(\omega_{1,2,3}) = \frac{1}{m(\omega_2+\omega_3)}\,\bar{\chi}_{n,\omega_1}(ig\mathcal{B}^\perp_\beta)_{\omega_2}(ig\mathcal{B}^\perp_\alpha)_{\omega_3}\Upsilon^{\alpha\beta}_{(h)}\mathcal{H}_v\,,$$

$$J^{(2i)}(\omega_{1,2,3}) = \frac{1}{m(\omega_2+\omega_3)}\mathrm{Tr}\big[(ig\mathcal{B}^\perp_\beta)_{\omega_2}(ig\mathcal{B}^\perp_\alpha)_{\omega_3}\big]\,\bar{\chi}_{n,\omega_1}\Upsilon^{\alpha\beta}_{(i)}\mathcal{H}_v\,.$$

$$J^{(2j)}(\omega_1,\omega_2,\omega_3) = \sum_{f=u,d,s}\big[\bar{\chi}^f_{n,\omega_2}\Upsilon_{(j\chi)}\chi^f_{n,\omega_3}\big]\big[\bar{\chi}_{n,\omega_1}\Upsilon_{(j\mathcal{H})}\mathcal{H}_v\big]\,,$$

$$J^{(2k)}(\omega_1,\omega_2,\omega_3) = \sum_{f=u,d,s}\big[\bar{\chi}^f_{n,\omega_2}T^A\Upsilon_{(k\chi)}\chi^f_{n,\omega_3}\big]\big[\bar{\chi}_{n,\omega_1}T^A\Upsilon_{(k\mathcal{H})}\mathcal{H}_v\big]$$

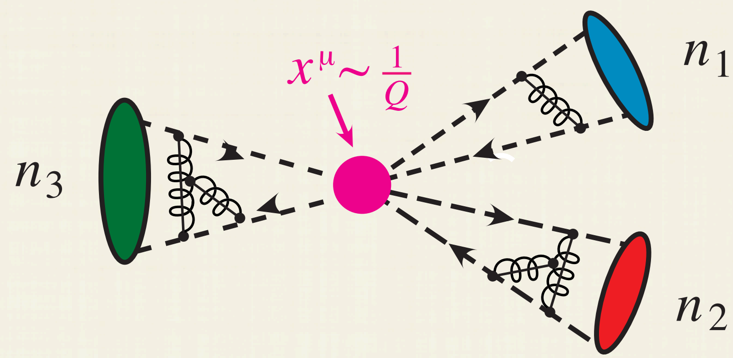
Heavy to light current up to NNLO

Connected

Arnesen, Kundu, Stewart hep-ph/0508214

Drawbacks

- it is hard to calculate
- if we have more *n* there is a set of RPI for each one



Is there a better way to find these constraints? YES!

If we found RPI OBJECTS we can construct currents with them

RPI IN HQET

Momentum of an heavy quark $P^\mu = mv^\mu + k^\mu$

It is invariant under $\begin{cases} v^\mu \to v^\mu + \beta^\mu \\ k^\mu \to k^\mu - m\beta^\mu \end{cases} \quad \beta^\mu \sim \lambda^2$

We can construct an invariant fermion field

$$H_v(x) = e^{-imv\cdot x}\left[\frac{1}{\sqrt{2}(1+v\cdot\mathcal{V}/|\mathcal{V}|)}\left(1+\frac{\not{\mathcal{V}}\,\not{v}}{|\mathcal{V}|}\right)\right]h_v(x) \qquad \mathcal{V}^\mu = v^\mu + iD^\mu_{\rm us}/m$$

Using this superfield gives operators that are invariant under RPI

Luke, Manohar hep-ph/9205228

OUR RPI OBJECTS FOR SCET

n-collinear Quark Field: $\psi_n = \left(1 + \frac{1}{\bar{n}\cdot D_n} \not{D}_n^{\perp} \frac{\bar{\not{n}}}{2}\right) \xi_n$

Gluon Field Strength: $ig\, G^n_{\mu\nu} = \left[iD^n_\mu, iD^n_\nu\right]$

Delta function: $\delta(\omega - 2q\cdot i\partial_n)$

RPI Wilson Line: $\mathcal{W}_n = W_n\, e^{-iR_n}$

- q^μ is a parameter of the process
- R_n is Hermitian, dimensionless, and collinear gauge invariant

$$R_n = R_n\left[\overline{\mathcal{P}}_n, \mathcal{P}^\mu_{n\perp}, in\cdot\partial, ig\mathcal{B}^\mu_{n\perp}, ign\cdot\mathcal{B}_n, q\right]$$

- these objects involve multiple orders in the power counting
- they are not gauge invariant objects
- we want to expand them in terms of χ_n , $ig\mathcal{B}^\mu_n$ ...

RPI AND GAUGE INVARIANT OBJECTS

- In SCET we use W_n to build gauge invariant objects

$$\chi_n \equiv W_n^\dagger \xi_n, \quad \mathcal{D}_n^\mu \equiv W_n^\dagger D_n^\mu W_n \,,$$

$$iD_n^{\perp\mu} = \mathcal{P}_{n\perp}^\mu + ig\mathcal{B}_{n\perp}^\mu \,, \qquad ig\mathcal{B}_{n\perp}^\mu \equiv \Big[\frac{1}{\overline{\mathcal{P}}_n}[i\bar{n}\cdot\mathcal{D}_n, i\mathcal{D}_n^{\perp\mu}]\Big] \,,$$

- It is useful to label the collinear part of the momentum $\bar{\mathcal{P}}_n$

$$\chi_{n,\omega} \equiv \Big[\delta(\omega - n\cdot q\,\overline{\mathcal{P}}_n)\chi_n\Big]$$

$$(ig\mathcal{B}^\mu)_\omega \equiv \Big[ig\mathcal{B}^\mu\delta(\omega - n\cdot q\overline{\mathcal{P}}_n^\dagger)\Big]$$

Similarly we defined the SUPERFIELDS

$$\Psi_{n,\omega} \equiv \Big[\delta(\omega - 2q\cdot i\partial_n)\mathcal{W}_n^\dagger\psi_n\Big]$$

$$\mathcal{G}_{n,\omega}^{\mu\nu} \equiv \Big[\frac{1}{\omega}\,\mathcal{W}_n^\dagger G_n^{\mu\nu}\mathcal{W}_n\,\delta(\omega - 2q\cdot i\overleftarrow{\partial}_n)\Big]$$

They are both RPI and gauge invariant (GI) objects

Also $i\partial^\mu = (n^\mu/2)\bar{\mathcal{P}} + \mathcal{P}_\perp^\mu + (\bar{n}^\mu/2)(in\cdot\partial)$ that acts on a gauge singlets is RPI and GI

RPI WILSON LINE

Collinear Wilson Line: $[(\bar{n}\cdot D)W_n]=0 \longrightarrow W_n=\left[\sum_{\mathbf{perms}} \mathbf{exp}\left(-g\frac{1}{\overline{\mathcal{P}}_n}\bar{n}\cdot A_n\right)\right]$

RPI Wilson Line: $[(q\cdot D)\mathcal{W}_n]=0 \longrightarrow \mathcal{W}_n=\left[\sum_{\mathbf{perms}} \mathbf{exp}\left(-g\frac{1}{(q\cdot i\partial)}q\cdot A_n\right)\right]$

iR_n connects W_n with RPI $\mathcal{W}_n$: $\mathcal{W}_n = W_n\, e^{-iR_n}$

We calculate iR_n in 3 steps

1. Use the relation $(q\cdot iD)=\mathcal{W}_n(q\cdot i\partial)\mathcal{W}_n^\dagger \longrightarrow (q\cdot i\mathcal{D})=e^{-iR_n}\,(q\cdot i\partial)\,e^{iR_n}$

2. B-C-H formula $(q\cdot i\mathcal{D})=(q\cdot i\partial)+\sum_{n=1}^{\infty}\frac{1}{n}\{(q\cdot i\partial),(iR_n)^n\}$ $\quad \{A,B^n\}=[[\cdots[A,\overbrace{B,]B,]\ldots,]B}^{n}]$

3. we λ expand $iR_n=\sum_{k=1}^{\infty} iR_n^{(k)}$ and solve order by order

$$\begin{cases} iR_n^{(1)} = \left[\dfrac{2}{n\cdot q\,\overline{\mathcal{P}}_n}\,q_\perp\cdot(ig\mathcal{B}_n^\perp)\right] \\[2ex] iR_n^{(2)}=\left[\dfrac{1}{n\cdot q\,\overline{\mathcal{P}}_n}\,(\bar{n}\cdot q)\,(nig\cdot\mathcal{B}_n)\right]-\left[\dfrac{4q_\perp\cdot\mathcal{P}_{n\perp}}{(n\cdot q\,\overline{\mathcal{P}}_n)^2}\,q_\perp\cdot(ig\mathcal{B}_n^\perp)\right]+\left[\dfrac{2}{n\cdot q\,\overline{\mathcal{P}}_n}\left[\left[\dfrac{1}{n\cdot q\,\overline{\mathcal{P}}_n}\,q_\perp\cdot(ig\mathcal{B}_n^\perp)\right],\,q_\perp\cdot(ig\mathcal{B}_n^\perp)\right]\right] \end{cases}$$

EXPANSION RPI OBJECTS IN λ

RPI Wilson line

$$\mathcal{W}_n = \sum_{k=0}^{\infty} \mathcal{W}_n^{(k)} = W_n e^{-iR_n} = \overset{\lambda^0}{W_n} - \overset{\lambda^1}{W_n (iR_n^{(1)})} + \overset{\lambda^2}{W_n \Big[\frac{1}{2}(iR_n^{(1)})^2 - (iR_n^{(2)})\Big]} + \ldots$$

δ function
(Taylor expansion)

$$\delta(\omega - 2q \cdot i\partial_n) = \delta(\omega - n\cdot q\,\bar{\mathcal{P}}_n - 2q_\perp\cdot\bar{\mathcal{P}}_{n\perp} - \bar{n}\cdot q\, in\cdot\partial)$$
$$= \Big(1 + \sum_{k=1}^{\infty} p_n^{(k)}\Big)\delta(\omega - n\cdot q\,\bar{\mathcal{P}}_n)$$

$$\begin{cases} p_n^{(1)} = -2q_\perp\cdot\mathcal{P}_{n\perp}\dfrac{d}{d\omega} \\ p_n^{(2)} = 2(q_\perp\cdot\mathcal{P}_{n\perp})^2\dfrac{d^2}{d\omega^2} - (\bar{n}\cdot q)(in\cdot\partial)\dfrac{d}{d\omega} \end{cases}$$

Superfields

$$\Psi_{n,\omega} \equiv \Big[\delta(\omega - 2q\cdot i\partial_n)\mathcal{W}_n^\dagger \psi_n\Big] = \sum_{k=1}^{\infty} \Psi_{n,\omega}^{(k)}$$

$$\begin{cases} \Psi_{n,\omega}^{(1)} = \chi_{n,\omega} \\ \Psi_{n,\omega}^{(2)} = \displaystyle\sum_{\omega_a} \frac{(n\cdot q)}{\omega} i\not{\mathcal{D}}^\perp_{n,\omega_a-\omega}\frac{\bar{\not{n}}}{2}\chi_{n,\omega_a} + \sum_{\omega_a} iR^{(1)}_{n,\omega-\omega_a}\chi_{n,\omega_a} + p_n^{(1)}\chi_{n,\omega} \end{cases}$$

$$\mathcal{G}^{\mu\nu}_{n,\omega} = \sum_{k=1}^{\infty} \mathcal{G}^{(k)\mu\nu}_{n,\omega}$$

$$\mathcal{G}^{(1)\mu\nu}_{n,\omega} = \frac{n^\nu}{2(n\cdot q)}(ig\mathcal{B}^\mu_{n\perp})_\omega - \frac{n^\mu}{2(n\cdot q)}(ig\mathcal{B}^\nu_{n\perp})_\omega$$

RPI EQUATION OF MOTION

Collinear Lagrangian

$$\mathcal{L}_q = \bar{\xi}_n \left(in \cdot D_n + i\not{D}_n^{\perp} \frac{1}{i\bar{n} \cdot D_n} i\not{D}_n^{\perp} \right) \frac{\not{\bar{n}}}{2} \xi_n = \bar{\psi}_n i\not{D}_n \psi_n \quad \longrightarrow \quad \not{D}_n \psi_n = 0$$

Equations of Motion

Multiply by $\mathcal{W}_n$ and insert $\mathcal{W}_n^{\dagger}\mathcal{W}_n$ we get the RPI EOM

$$\hat{\not{\mathcal{D}}}_n \Psi_n = 0$$

where $\hat{\mathcal{D}}_n^{\mu}$ is gauge invariant and covariant derivative: $\hat{\mathcal{D}}_n^{\mu} \equiv e^{iR_n} \mathcal{D}_n^{\mu} e^{-iR_n}$

With some algebra... $\boxed{i\not{\partial}\Psi_n = q_\mu \gamma^\nu \mathcal{G}_n^{\mu\nu} \Psi_n}$

EOM for $\mathcal{G}_n^{\mu\nu}$ $\boxed{[(q \cdot i\partial) i\partial_\nu \mathcal{G}_n^{\nu\mu}] = g^2 T^A \bar{\Psi}_n T^A \gamma^\mu \Psi_n + [q_\alpha \mathcal{G}_{n\,\nu}^{\alpha}, (q \cdot i\partial)\mathcal{G}_n^{\mu\nu}]}$

Bianchi Identity $\boxed{i\not{\partial}\mathcal{G}_n^{\mu\nu} = \gamma^\alpha q^\beta \left(\mathcal{G}_n^{\alpha\beta}\mathcal{G}_n^{\mu\nu} - \mathcal{G}_n^{\beta[\mu}\mathcal{G}_n^{\nu]\alpha} \right) - \gamma_\alpha i\partial^{[\mu}\mathcal{G}_n^{\nu]\alpha}}$

GI BASIS

$$\begin{aligned}
O^{(0)} &= \bar\chi_{n_1,\omega_1}\Xi\chi_{n_2,\omega_2}\\
O^{(1a)} &= \bar\chi_{n_1,\omega_1}\Xi^{\alpha}\mathcal{P}^{\perp\dagger}_{n_1\alpha}\chi_{n_2,\omega_2}\\
O^{(1b)} &= \bar\chi_{n_1,\omega_1}\Xi^{\alpha}\mathcal{P}^{\perp}_{n_2,\alpha}\chi_{n_2,\omega_2}\\
O^{(1c)} &= \bar\chi_{n_1,\omega_1}\Xi^{\beta}(ig\mathcal{B}^{\perp}_{n_3\beta})_{\omega_3}\chi_{n_2,\omega_2}
\end{aligned}$$

RPI and GI BASIS

$$\begin{aligned}
\mathbf{O}^{(0a)} &= \bar\Psi_{n_1,\omega_1}\Gamma_{(a)}\Psi_{n_2,\omega_2}\\
\mathbf{O}^{(0b)} &= \bar\Psi_{n_1,\omega_1}\Gamma^{\alpha}_{(b)}i\partial_{n_2,\alpha}\Psi_{n_2,\omega_2}\\
\mathbf{O}^{(0c)} &= \bar\Psi_{n_1,\omega_1}\Gamma^{\alpha}_{(c)}(-i\overleftarrow{\partial}_{n_1,\alpha})\Psi_{n_2,\omega_2}\\
\mathbf{O}^{(1a)} &= \bar\Psi_{n_1,\omega_1}\Theta_{(a)\beta\beta'}\mathcal{G}^{\beta\beta'}_{n_3,\omega_3}\Psi_{n_2,\omega_2}
\end{aligned}$$

$$S = \sum_i C_i O_i$$

$$S = \sum_i \mathbf{C}_i \mathbf{O}_i$$

Reparametrization invariant $\delta_{RP} S = 0$

$$\delta_{RP} S = \delta_{RP} \sum_i C_i O_i = 0$$

$$\delta_{RP} S = \delta_{RP} \sum_i \mathbf{C}_i \mathbf{O}_i = \sum_i \mathbf{C}_i \delta_{RP} \mathbf{O}_i = 0$$

independent Wilson coefficients = # number of independent RPI operators

Using the RPI basis we find right away all the constraints

Scalar chiral-even up to NLO operator in GI basis

$$S(q) = C\ (n\cdot q)\mathcal{J}_V + \sum_{i=1}^{2} D_i\,(q_\alpha \mathcal{V}_i^\alpha) + \sum_{i=1}^{2} \tilde{D}_i\,(q_\alpha \tilde{\mathcal{V}}_i^\alpha) + E\ (q_\alpha \mathcal{V}_3^\alpha)$$

Scalar chiral-even up to NLO operator in RPI basis

$$\mathbf{O}^{(0)} = \bar{\Psi}_{n,\omega_1} \not{q} \Psi_{n,\omega_2}$$

there is only one RPI operator $\longrightarrow$ there is only one independent Wilson coefficient

$$\begin{aligned}
S(q) =& \mathbf{C}(\omega_1,\omega_2)\bar{\Psi}_{n,\omega_1}\not{q}\Psi_{n,\omega_2} \\
=& \mathbf{C}(\omega_1,\omega_2)\bar{\chi}_{n,\omega_1}\frac{\bar{\not{n}}}{2}(n\cdot q)\chi_{n,\omega_2} \\
& - \mathbf{C}(\omega_1,\omega_2)\left(\bar{\chi}_n \frac{\bar{\not{n}}}{2} i\overleftarrow{\not{\mathcal{D}}}_\perp \frac{1}{\bar{\mathcal{P}}^\dagger}\right)_{\omega_1} \not{q}_\perp \chi_{n,\omega_b} + \mathbf{C}(\omega_1,\omega_2)\bar{\chi}_{\omega_1}\not{q}_\perp \left(\frac{1}{\bar{\mathcal{P}}} i\not{\mathcal{D}}_\perp \frac{\bar{\not{n}}}{2}\chi_n\right)_{\omega_2} \\
& - 2\mathbf{C}(\omega_1,\omega_2)\left(\bar{\chi}_n\left[\mathcal{B}_\perp\cdot q_\perp \frac{1}{(n\cdot q)\bar{\mathcal{P}}^\dagger}\right]\right)_{\omega_1}\frac{\bar{\not{n}}}{2}(n\cdot q)\chi_{n,\omega_b} \\
& - 2\mathbf{C}(\omega_1,\omega_2)\bar{\chi}_{\omega_1}\frac{\bar{\not{n}}}{2}(n\cdot q)\left(\left[\frac{1}{(n\cdot q)\bar{\mathcal{P}}}\mathcal{B}_\perp\cdot q_\perp\right]\chi_n\right)_{\omega_2} \\
& + 2\frac{\partial \mathbf{C}(\omega_1,\omega_2)}{\partial\omega_1}\bar{\chi}_{n,\omega_1}\mathcal{P}_\perp^\dagger\cdot q_\perp \frac{\bar{\not{n}}}{2}(n\cdot q)\chi_{n,\omega_2} + 2\frac{\partial \mathbf{C}(\omega_1,\omega_2)}{\partial\omega_2}\bar{\chi}_{n,\omega_1}\frac{\bar{\not{n}}}{2}(n\cdot q)\mathcal{P}_\perp\cdot q_\perp\chi_{n,\omega_2}
\end{aligned}$$

n-$\bar{n}$ CURRENTS AT NLO

Important in study 2 jets events where at LO the main operators are

$$J(\omega_{1,2}) = \bar{\chi}_{\bar{n},\omega_1}\Gamma^\mu \chi_{n,\omega_2} \qquad \Gamma^\mu = \{\gamma_\perp^\mu, \gamma_\perp^\mu\gamma_5\}$$

GI basis

$$\begin{aligned}
J_1(\omega_{1,2}) =& \bar{\chi}_{\bar{n},\omega_1}\gamma_\perp^\mu \chi_{n,\omega_2}\\
J_2(\omega_{1,3,2}) =& \bar{\chi}_{\bar{n},\omega_1} n^\mu (ig\mathcal{B}_n^\perp)_{\omega_3}\chi_{n,\omega_2}\\
J_3(\omega_{1,3,2}) =& \bar{\chi}_{\bar{n},\omega_1} n^\mu (ig\mathcal{B}_{\bar{n}}^\perp)_{\omega_3}\chi_{n,\omega_2}\\
J_4(\omega_{1,3,2}) =& \bar{\chi}_{\bar{n},\omega_1} \bar{n}^\mu (ig\mathcal{B}_n^\perp)_{\omega_3}\chi_{n,\omega_2}\\
J_5(\omega_{1,3,2}) =& \bar{\chi}_{\bar{n},\omega_1} \bar{n}^\mu (ig\mathcal{B}_{\bar{n}}^\perp)_{\omega_3}\chi_{n,\omega_2}
\end{aligned}$$

RPI basis

$$\begin{aligned}
\mathbf{J}_1^{(0)} &= \bar{\Psi}_{\bar{n},\omega_1}\gamma^\mu \Psi_{n,\omega_2}\\
\mathbf{J}_1^{(1)} &= \bar{\Psi}_{\bar{n},\omega_1} q^\mu \not{q}\, \Psi_{n,\omega_2}\\
\mathbf{J}_2^{(1)} &= \bar{\Psi}_{\bar{n},\omega_1} \not{q}\, i\partial_n^\mu \Psi_{n,\omega_2}\\
\mathbf{J}_3^{(1)} &= \bar{\Psi}_{\bar{n},\omega_1} \gamma_\beta \mathcal{G}_{n,\omega_3}^{\mu\beta} \Psi_{n,\omega_2}\\
\mathbf{J}_4^{(1)} &= \bar{\Psi}_{\bar{n},\omega_1} \gamma_\beta \mathcal{G}_{\bar{n},\omega_3}^{\mu\beta} \Psi_{n,\omega_2}\\
\mathbf{J}_5^{(1)} &= \bar{\Psi}_{\bar{n},\omega_1} q^\mu \gamma_\alpha q_\beta \mathcal{G}_{n,\omega_3}^{\alpha\beta} \Psi_{n,\omega_2}\\
\mathbf{J}_6^{(1)} &= \bar{\Psi}_{\bar{n},\omega_1} q^\mu \gamma_\alpha q_\beta \mathcal{G}_{\bar{n},\omega_3}^{\alpha\beta} \Psi_{n,\omega_2}\\
\mathbf{J}_6^{(1)} &= \bar{\Psi}_{\bar{n},\omega_1} q^\mu \gamma_\alpha q_\beta \mathcal{G}_{\bar{n},\omega_3}^{\alpha\beta} \Psi_{n,\omega_2}\\
\mathbf{J}_7^{(1)} &= \bar{\Psi}_{\bar{n},\omega_1} q^\mu \not{q}\gamma_\alpha \gamma_\beta \mathcal{G}_{\bar{n},\omega_3}^{\alpha\beta} \Psi_{n,\omega_2}\\
\mathbf{J}_8^{(1)} &= \bar{\Psi}_{\bar{n},\omega_1} \gamma^\mu \gamma_\alpha \gamma_\beta \mathcal{G}_{\bar{n},\omega_3}^{\alpha\beta} \Psi_{n,\omega_2}\\
\mathbf{J}_9^{(1)} &= \bar{\Psi}_{\bar{n},\omega_1} \gamma_\alpha q_\beta i\partial_n^\mu \mathcal{G}_{n,\omega_3}^{\alpha\beta} \Psi_{n,\omega_2}\\
\mathbf{J}_{10}^{(1)} &= \bar{\Psi}_{\bar{n},\omega_1} \gamma_\alpha q_\beta i\overleftarrow{\partial}_{\bar{n}}^\mu \mathcal{G}_{\bar{n},\omega_3}^{\alpha\beta} \Psi_{n,\omega_2}\\
\mathbf{J}_{11}^{(1)} &= \bar{\Psi}_{\bar{n},\omega_1} \gamma_\alpha q_\beta \left[i\partial_{\bar{n}}^\mu \mathcal{G}_{\bar{n},\omega_3}^{\alpha\beta}\right] \Psi_{n,\omega_2}\\
\mathbf{J}_{12}^{(1)} &= \bar{\Psi}_{\bar{n},\omega_1} \not{q}\gamma_\alpha \gamma_\beta i\overleftarrow{\partial}_{\bar{n}}^\mu \mathcal{G}_{\bar{n},\omega_3}^{\alpha\beta} \Psi_{n,\omega_2}\\
\mathbf{J}_{13}^{(1)} &= \bar{\Psi}_{\bar{n},\omega_1} \not{q}\gamma_\alpha \gamma_\beta \left[i\partial_{\bar{n}}^\mu \mathcal{G}_{\bar{n},\omega_3}^{\alpha\beta}\right] \Psi_{n,\omega_2}
\end{aligned}$$

- as q we take the momentum transfer from the virtual photon
- frame where $q_\perp = 0$
- The RPI basis is overcounted
- NO connections
- Also at NNLO there are no connections

CONCLUSIONS

- We constructed a set of RPI objects in SCET
- Using them it is easy to see if there are connections coming from RPI
- In $n-\bar{n}$ vector currents we proved that there are no connections at NLO and NNLO