Top Mass from Jets

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Based on work with: Sean Fleming, Andre Hoang, & Iain Stewart
(hep-ph/0703207, more in preparation)

Motivation

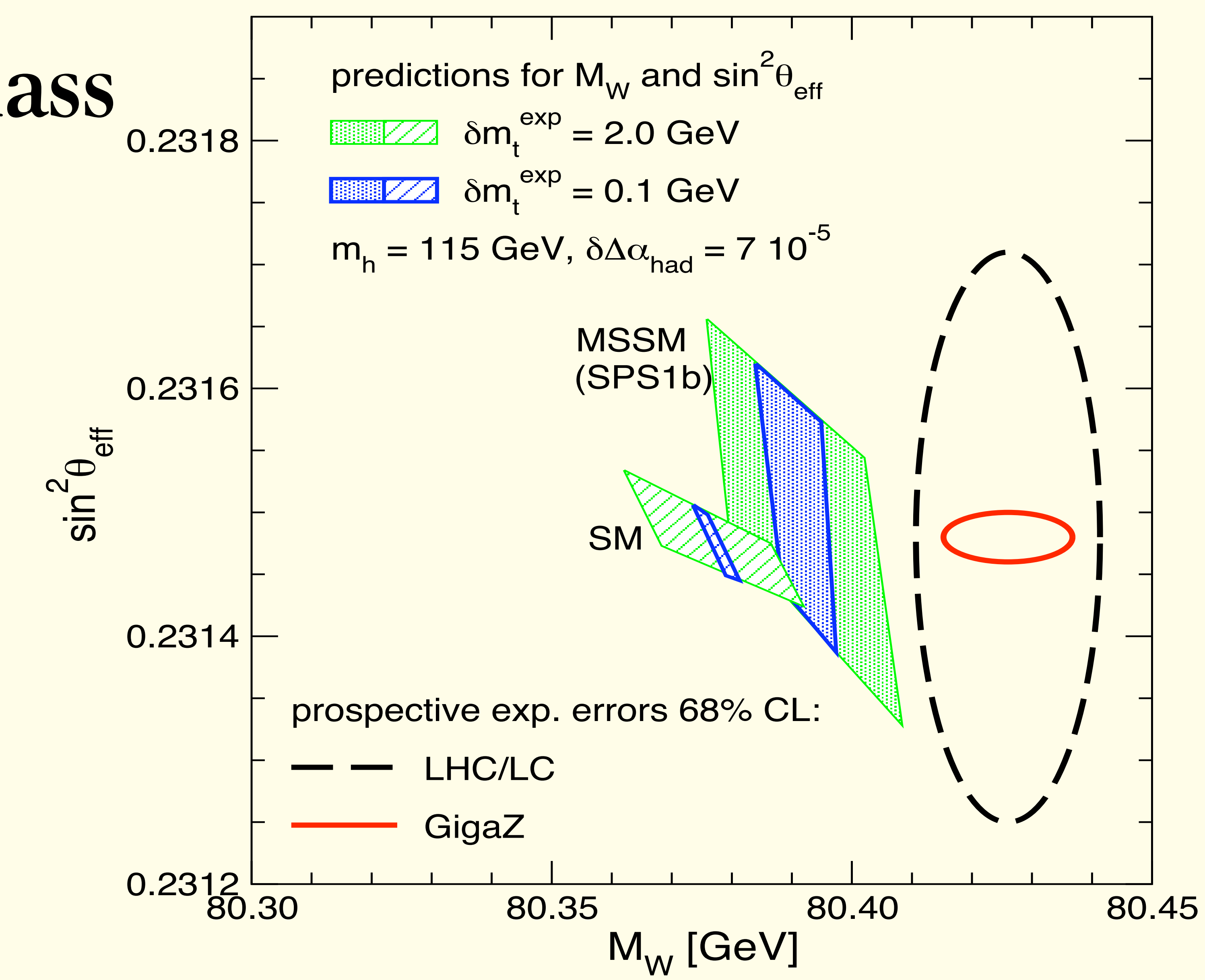
- Top quark couples strongly to the Higgs sector and a good probe of new physics.

- The top mass is the dominant source of theoretical uncertainty in EWPOs.

- Typically the uncertainty in the extracted Higgs mass will be limited by the uncertainty in the top mass.

$$\delta m_t \sim \delta m_h.$$

- Good reasons to measure the top mass with high precision.

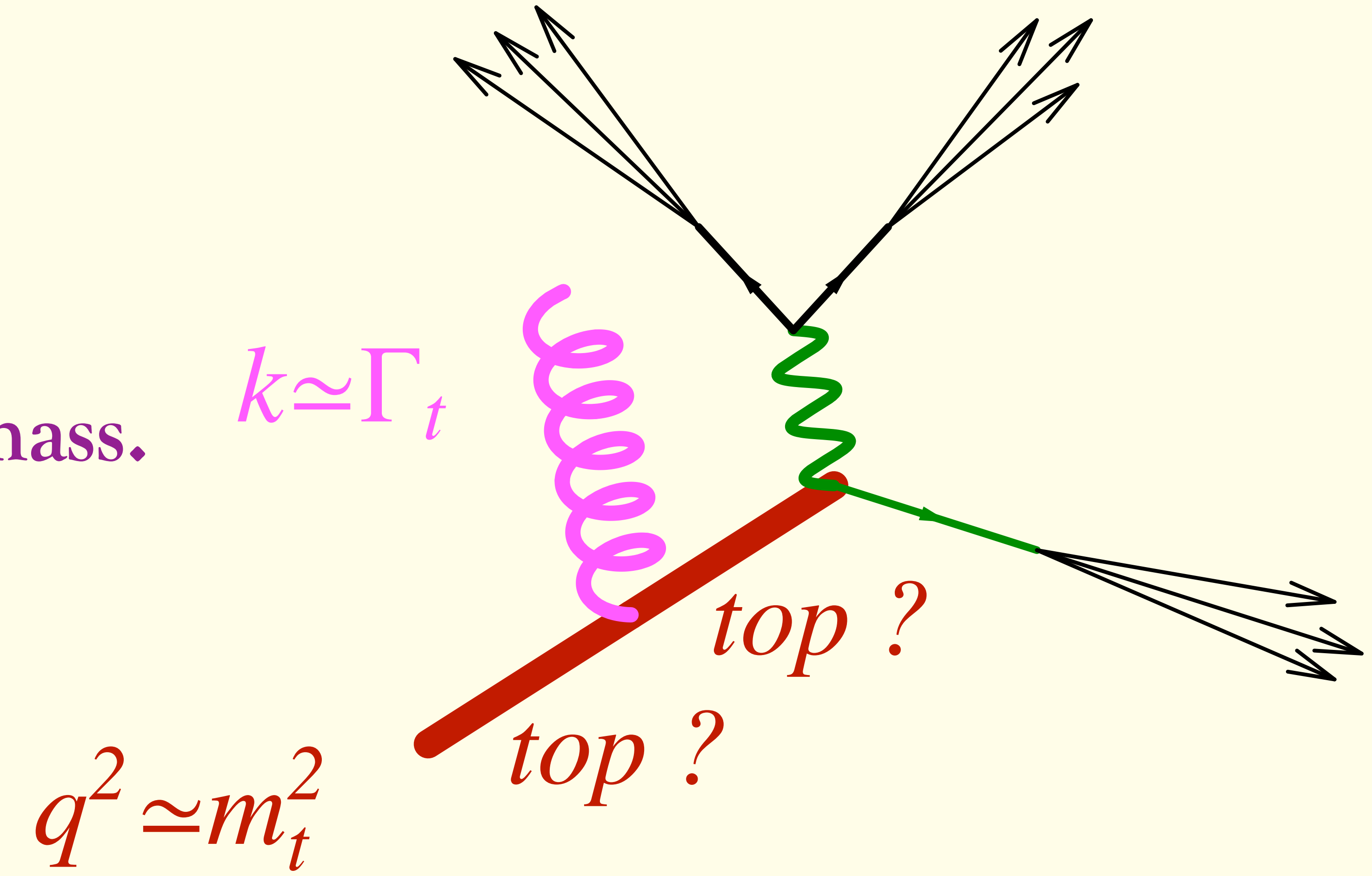


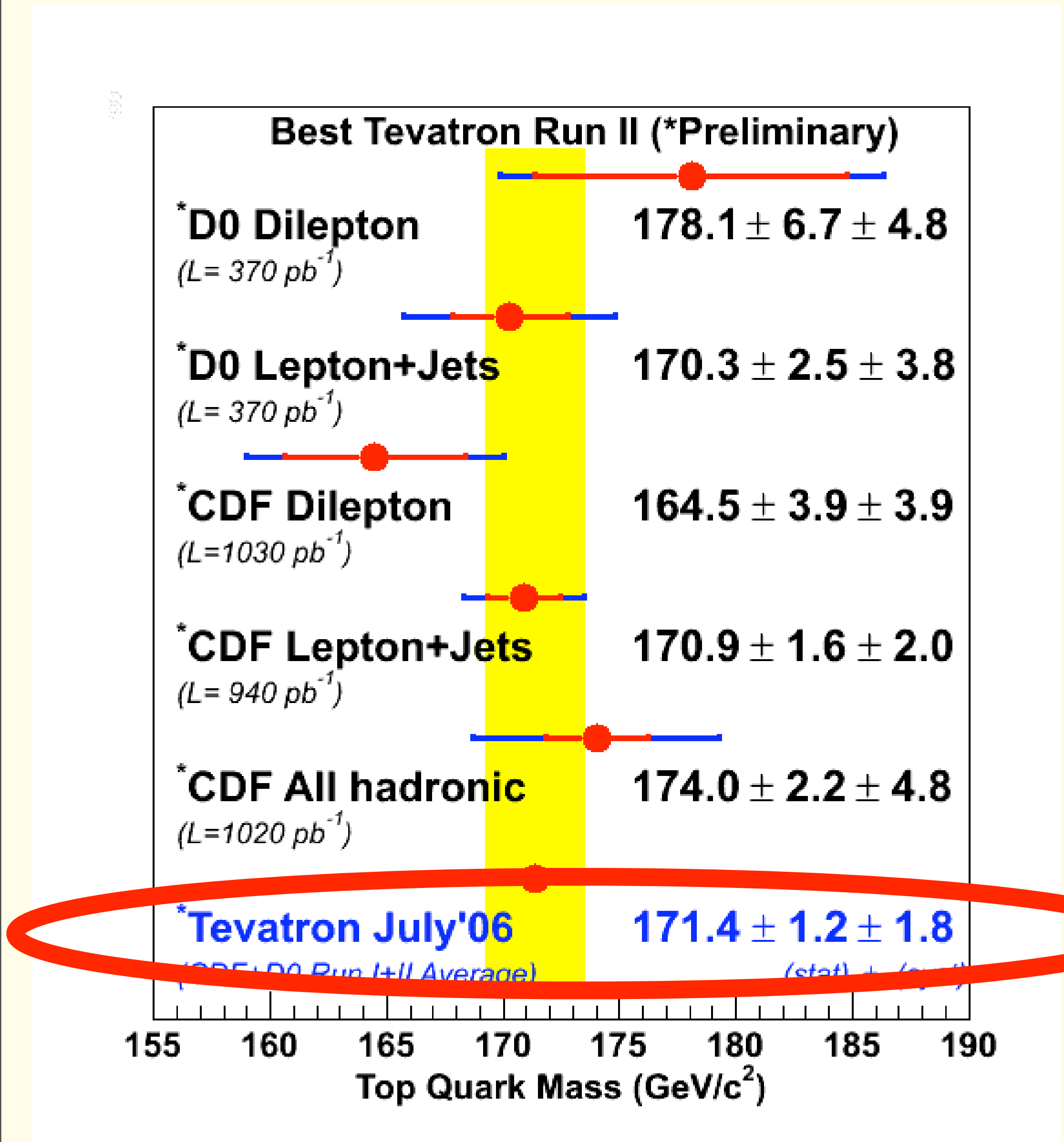
Predictions for the W mass and Weinberg Angle in the SM & MSSM

What are we Measuring?

- What is the top mass?

  - Top is a colored parton. Cannot define physical on-shell mass.
  - Top mass is a parameter of the Lagrangian.
  - Top mass parameter is scheme dependent.





Current Top Mass Mesurements

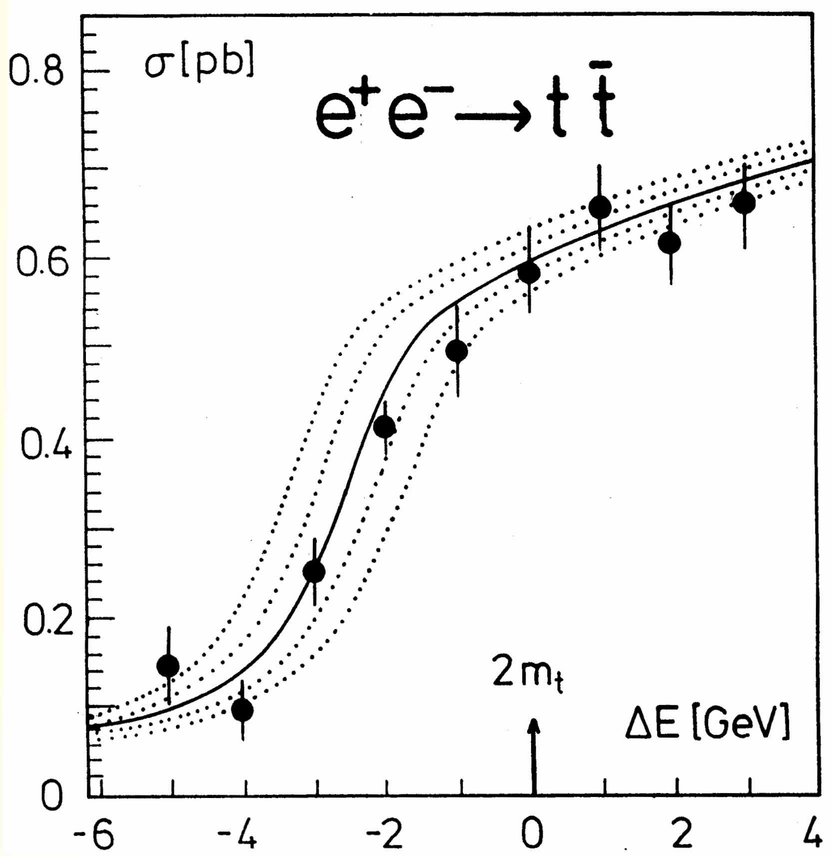
- Which top mass?

  - Which mass are the experimentalists measuring?
  - Pole mass? : $\delta m \sim \Lambda_{\mathrm{QCD}}$ renormalon ambiguity, poor perturbative behavior.
  - For better precision we need a short distance top mass.
  - How can we extract a short distance mass? Which mass?

Observables

- What is a suitable top mass observable?
  - Clear and well defined relation to a short distance mass.
  - Good signal to background ratio.

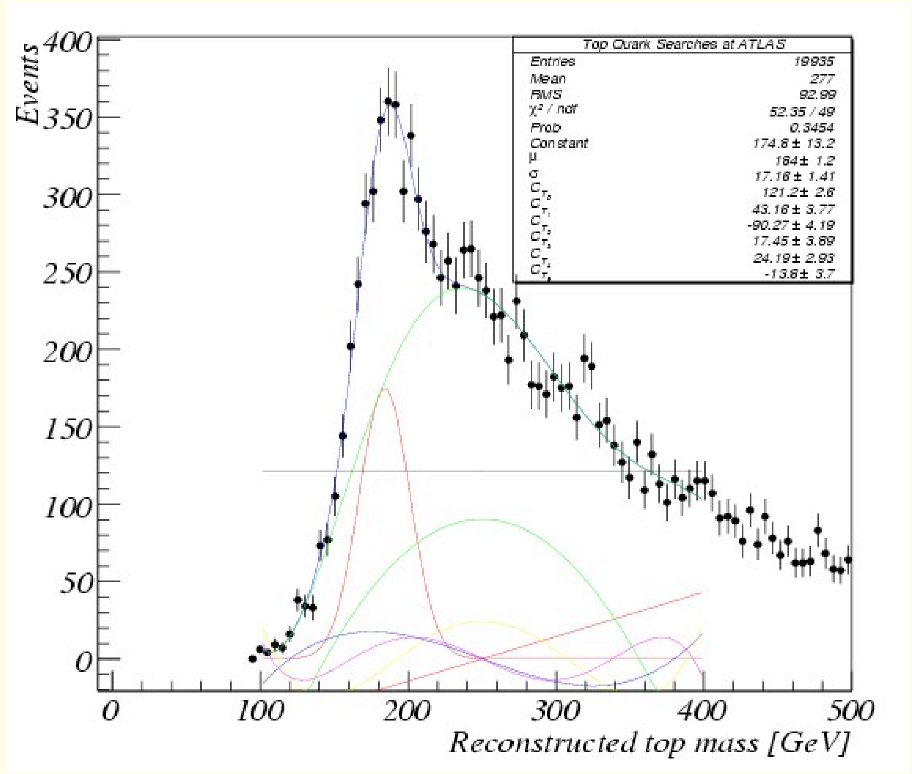
Threshold Scan



- Physics well understood
  - NRQCD is the appropriate EFT.
  - Well defined relation to short distance mass.
  - Backgrounds well understood.

$\delta m_t^{th} \sim 100 MeV$ *(Hoang, Manohar, Stewart, Teubner,...)*

Jet Reconstruction



- Many open theoretical & experimental questions
  - Relation to short distance mass.
  - Backgrounds,...

Jet Reconstruction Issues

- Suitable jet observable with clear relation to a short distance mass. ★
- Final state soft radiation. ★
- Initial state soft radiation.
- Initial state PDFs.
- Jet Energy Scale. ★
- Beam Remnants.
- Underlying Events.
- ....

★ Issues common to the ILC & LHC

Pair Production of Top Jets

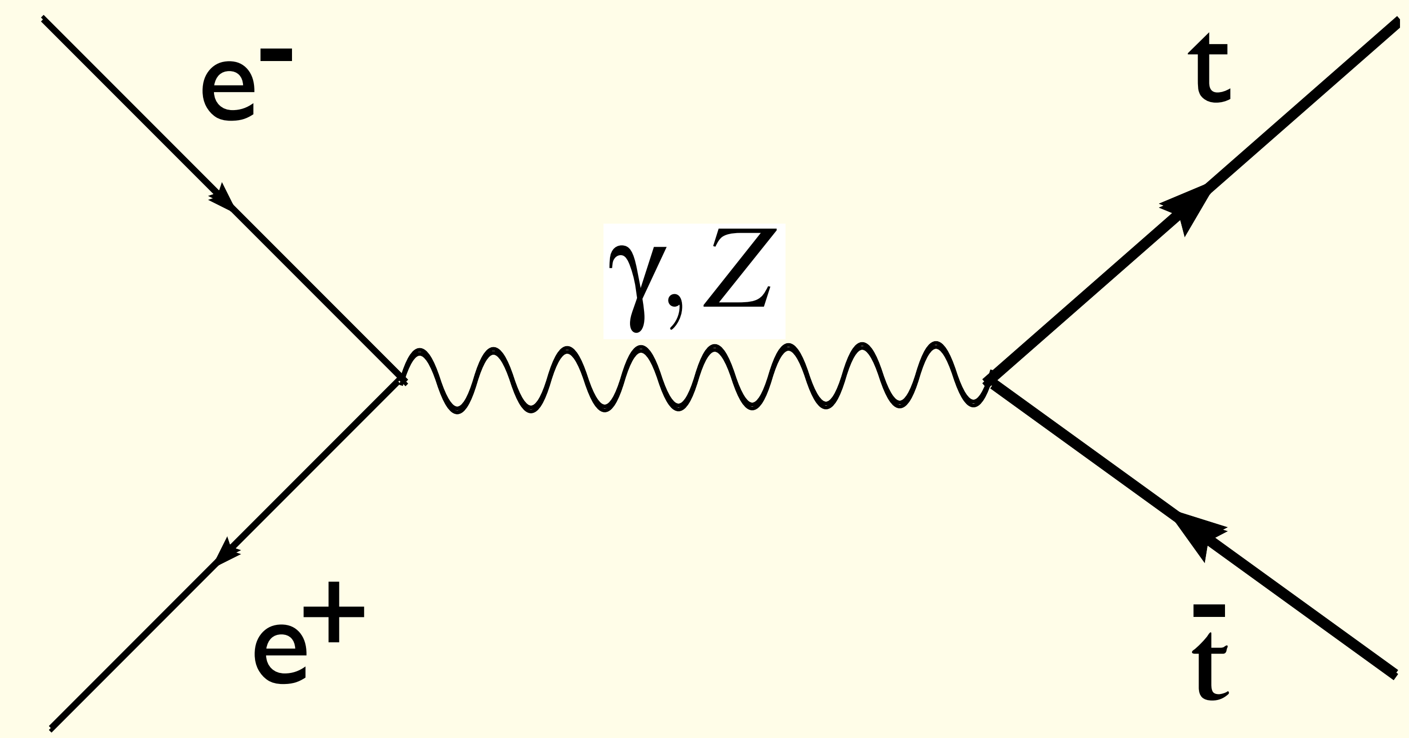
$$e^+e^- \longrightarrow t\bar{t} \qquad pp \rightarrow t\bar{t}X$$

LC

LHC

Focus of this talk

Jet Observable Sensitive to Top Mass



- Focus on the dijet region where the top and antitop jets have invariant masses close to the top mass.

- The top and antitop jets are defined to have the invariant masses:

$$M_t\,,\;M_{\bar t}$$

- The jet invariant mass condition is characterized as:

$$\hat{s}_{t,\bar t} \equiv \frac{s_{t,\bar t}}{m} \equiv \frac{M^2_{t,\bar t} - m^2}{m} \sim \Gamma \ll m$$

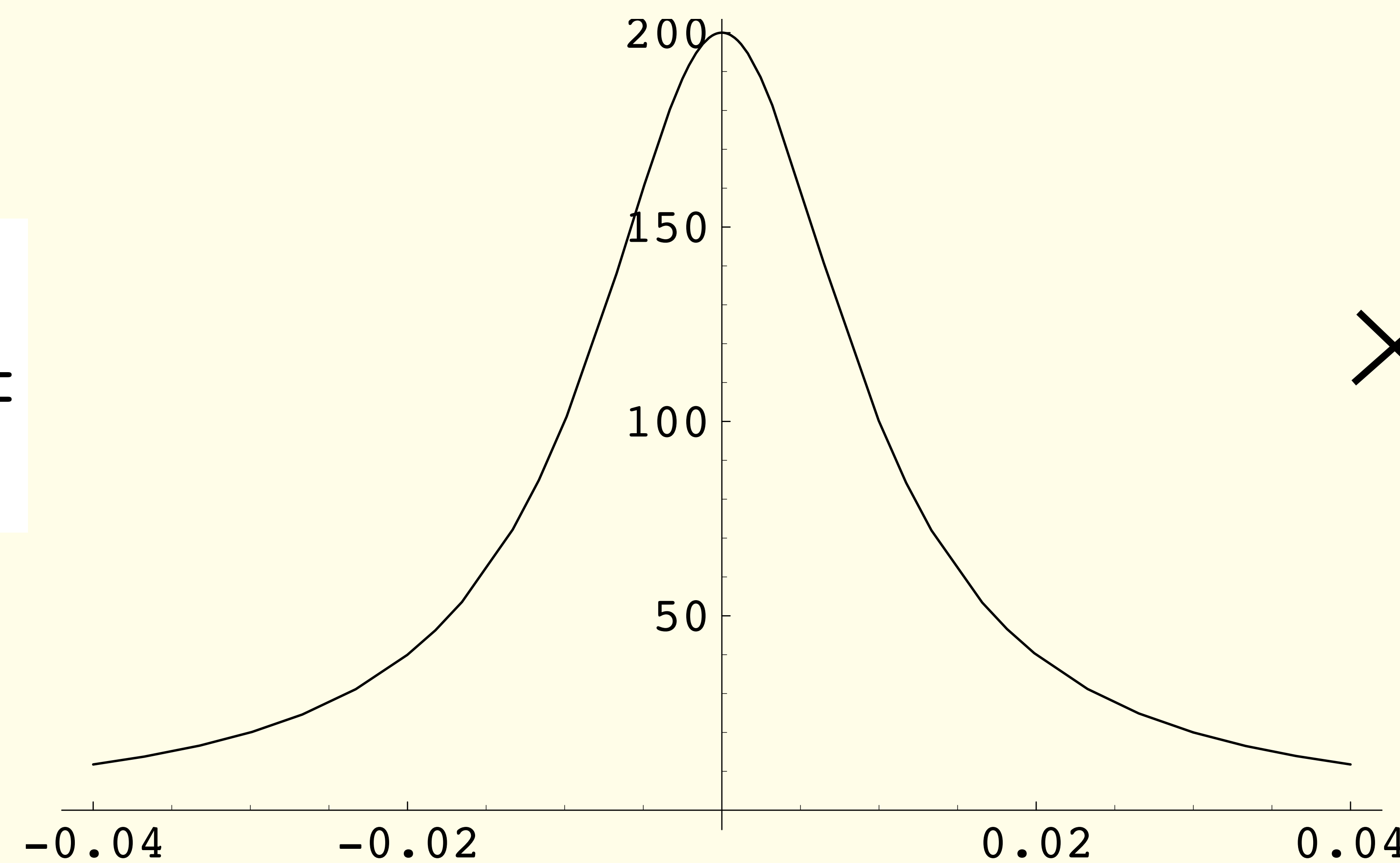
- The jet observable of interest is the double differential jet invariant mass distribution:

$$\frac{d\sigma}{dM_t^2\, dM_{\bar t}^2}$$

Tree Level Breit Wigner Curves?

- A first guess might be that the distribution is a product of Breit Wigner curves.

$$\frac{d^2\sigma}{ds_t\, ds_{\bar{t}}}\bigg|_{\text{tree level}} =$$



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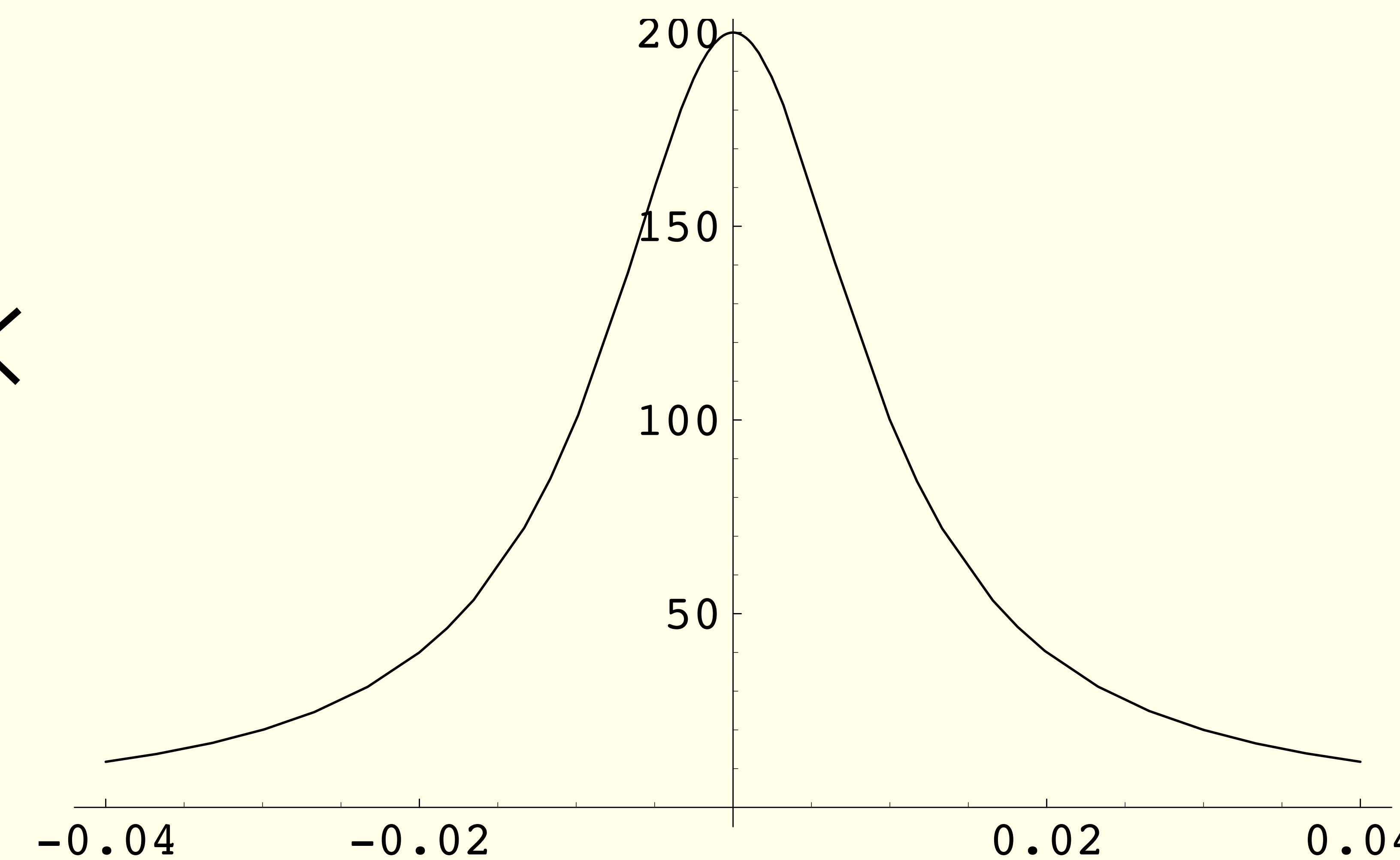


- We will find that this is not always true even at tree level due to nonperturbative effects.

- Furthermore large logarithms can affect these curves.

Relevant Energy Scales

- Center of mass energy $Q \sim 1\mathrm{TeV}$
- Top quark mass $m \sim 174\mathrm{GeV}$
- Top quark width $\Gamma \sim 2\mathrm{GeV}$
- Confinement Scale $\Lambda \sim 500 MeV$

Disparate energy scales $\longrightarrow$ Effective Field Theory!

Effective Field Theories

Kinematics for Top Jets: I

- **High Energy Condition: Top quark pairs are produced with a center of mass energy much larger than the top mass**

$$Q \gg m$$

- **In this limit one can treat top quarks as collinear degrees of freedom in the Soft Collinear Effective Theory (SCET)** *(Bauer, Fleming, Luke, Pirjol, Stewart)*.

Kinematics for Top Jets: II

- Invariant Mass Condition: We characterize on shell production by the requirement:

$$M_{t,\bar{t}}^2 - m^2 \lesssim m\Gamma$$

- This condition looks like the invariant mass constraint on a heavy quark in Heavy Quark Effective Theory (HQET) *(Isgur, Wise,...)*.

- HQET has been generalized to unstable particles *(Beneke, Chapovsky, Signer, Zanderighi)*.

Group Photo of Effective Field Theories

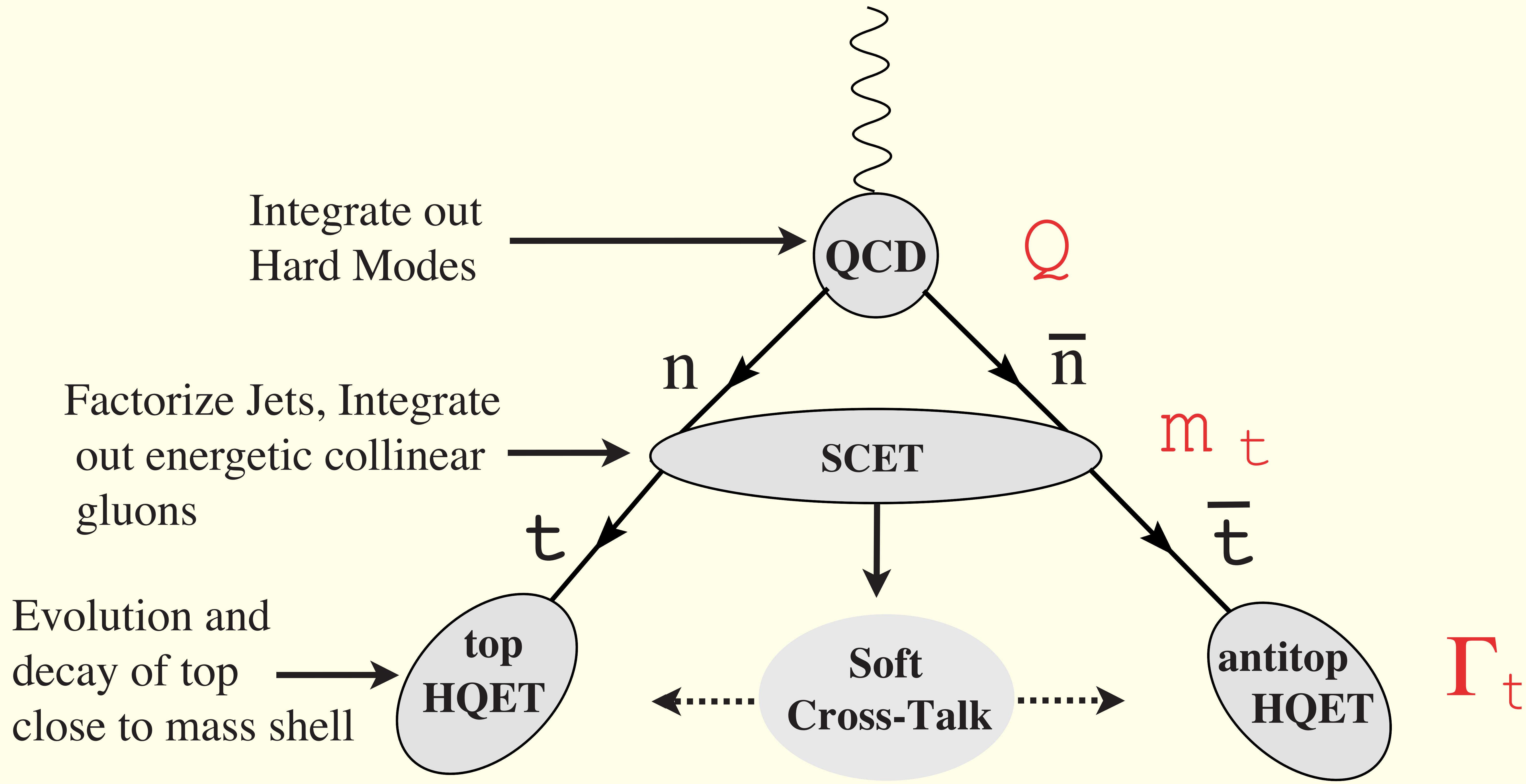
QCD Q

↓

SCET m

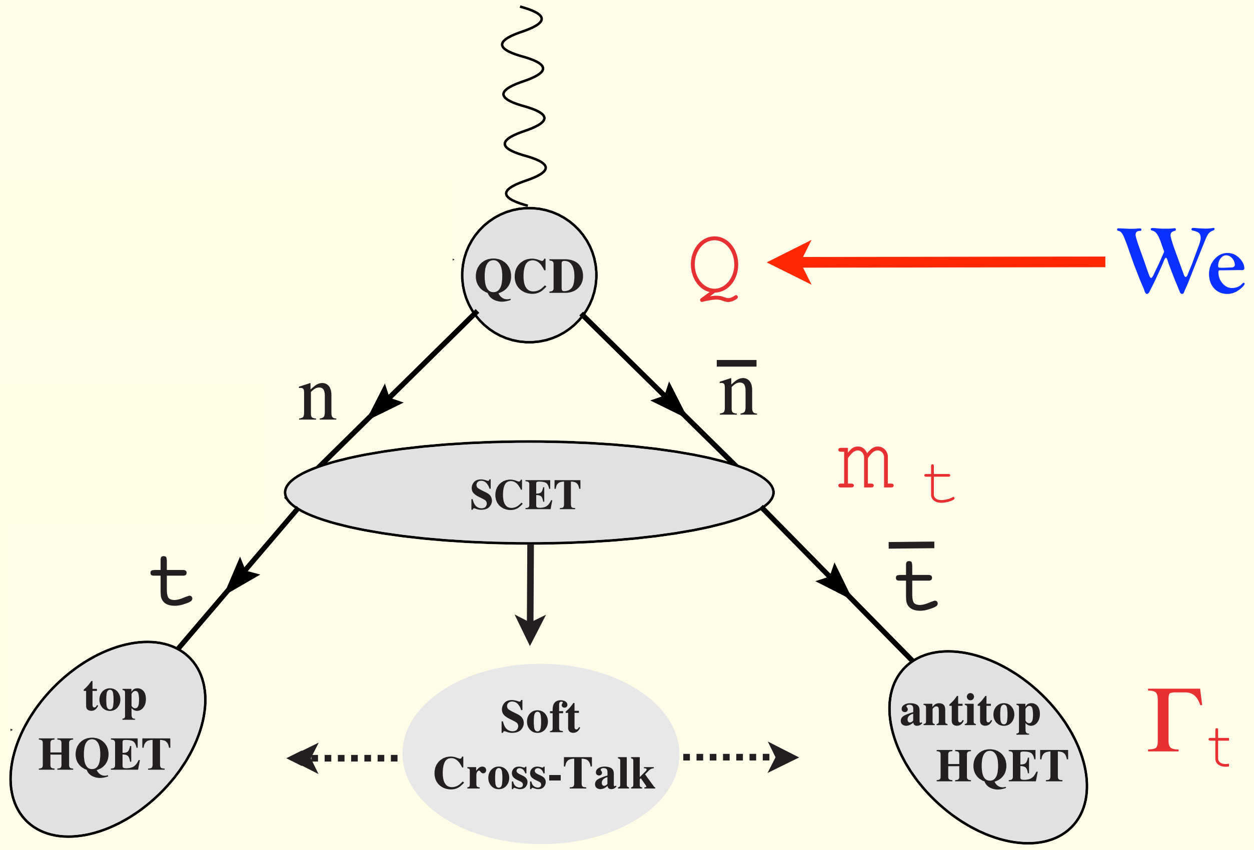
↓

HQET Γ



The QCD Cross-Section

The QCD Cross-Section



- The cross-section in QCD has the general form :

$$\sigma = \sum_{X}^{res.} (2\pi)^4 \, \delta^4(p_e + p_{\bar{e}} - p_X) \sum_{ij} L_{\mu\nu}^{(ij)} \, \langle 0 | J_i^\mu(0) | X \rangle \langle X | J_j^{\dagger\nu}(0) | 0 \rangle$$

- The sum over final states X is restricted to contain a top jet and an anti-top jet with invariant masses close to the top mass.

- The top quark currents are produced by photon and Z exchange:

$$J_i^\mu(x) = \bar{\psi}(x) \Gamma_i^\mu \psi(x) \,, \quad \Gamma_\gamma^\mu = \gamma^\mu , \, \Gamma_Z^\mu = g^V \gamma^\mu + g^A \gamma^\mu \gamma_5$$

Matching QCD Current onto SCET

- We restrict the final state phase space to high energy top quark pairs by matching the QCD current onto the SCET current :

$$J_i^\mu(0) = \int d\omega\, d\bar{\omega}\, C(\omega,\bar{\omega},\mu)\mathcal{J}_i^\mu(\omega,\bar{\omega},\mu)$$

QCD Wilson Coeff. SCET

$$\mathcal{J}_i^\mu(\omega,\bar{\omega},\mu) = \bar{\chi}_{n,\omega}(0)\Gamma_i^\mu\chi_{\bar{n},\bar{\omega}}(0)\ , \qquad \chi_{n,\omega}(0) = \delta(\omega-\bar{\mathcal{P}})(W^\dagger\xi_n)(0)$$

Jet field

- By momentum conservation, the relevant Wilson coefficient that survives is

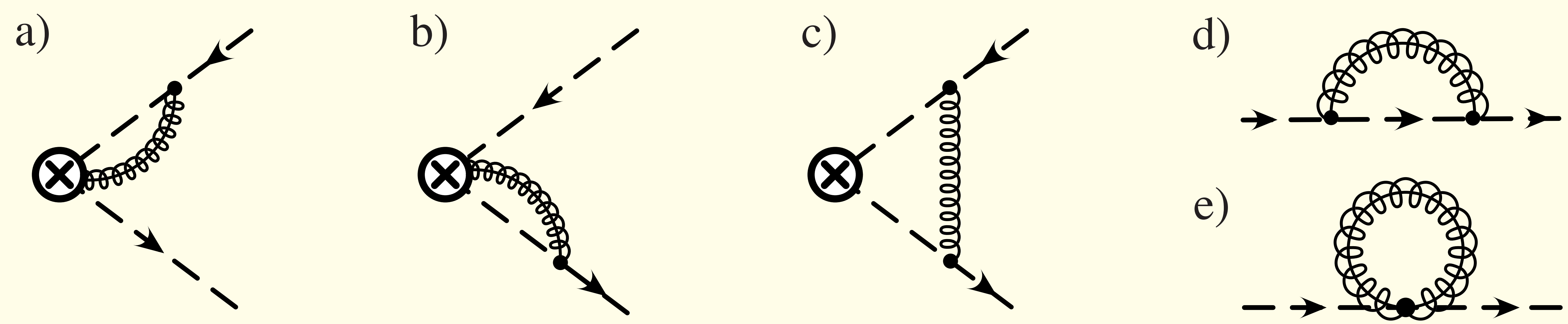
$$C(-Q,Q,\mu) \equiv C(Q,\mu)$$

- In this step of matching, the hard modes of QCD are integrated out.

Matching QCD onto SCET at One Loop

QCD

SCET



$$C(\mu) = 1 + \frac{\alpha_s C_F}{4\pi}\left[3\ln\frac{-Q^2}{\mu^2} - \ln^2\frac{-Q^2}{\mu^2} - 8 + \frac{\pi^2}{6}\right]$$

- Note that the logs in the Wilson coefficient vanish by choosing the matching scale at :

$$\mu = Q$$

The SCET Cross-Section

The SCET Cross-Section

- After matching the QCD current onto SCET, the cross-section has the general form :

$$\sigma = \sum_{\vec{n}} \sum_{X_n X_{\bar{n}} X_s}^{res.} (2\pi)^4 \, \delta^4(q - P_{X_n} - P_{X_{\bar{n}}} - P_{X_s}) \sum_i L_{\mu\nu}^{(i)} \int d\omega \, d\bar{\omega} \, d\omega' \, d\bar{\omega}'$$
$$\times C(\omega, \bar{\omega}) C^*(\omega', \bar{\omega}') \langle 0 | \bar{\chi}_{\bar{n},\bar{\omega}'} \bar{\Gamma}_j^{\nu} \chi_{n,\omega'} | X_n X_{\bar{n}} X_s \rangle \langle X_n X_{\bar{n}} X_s | \overline{\chi}_{n,\omega} \Gamma_i^{\mu} \chi_{\bar{n},\bar{\omega}} | 0 \rangle$$

- The complete set of states in SCET involve only soft and collinear degrees of freedom.

$$|X\rangle = |X_n X_{\bar{n}} X_s\rangle = \underbrace{|X_n\rangle}_{\text{Collinear: } n} \otimes \underbrace{|X_{\bar{n}}\rangle}_{\text{Collinear: } \bar{n}} \otimes \underbrace{|X_s\rangle}_{\text{Soft}}$$

Factorized Cross Section in SCET

soft

$$\sigma = K_0 \sum_{\vec{n}} \sum_{X_n X_{\bar{n}} X_s}^{res.} (2\pi)^4 \, \delta^4(q - P_{X_n} - P_{X_{\bar{n}}} - P_{X_s}) \langle 0|\overline{Y}_{\bar{n}} \, Y_n|X_s\rangle\langle X_s|Y_n^\dagger \, \overline{Y}_{\bar{n}}^\dagger|0\rangle$$

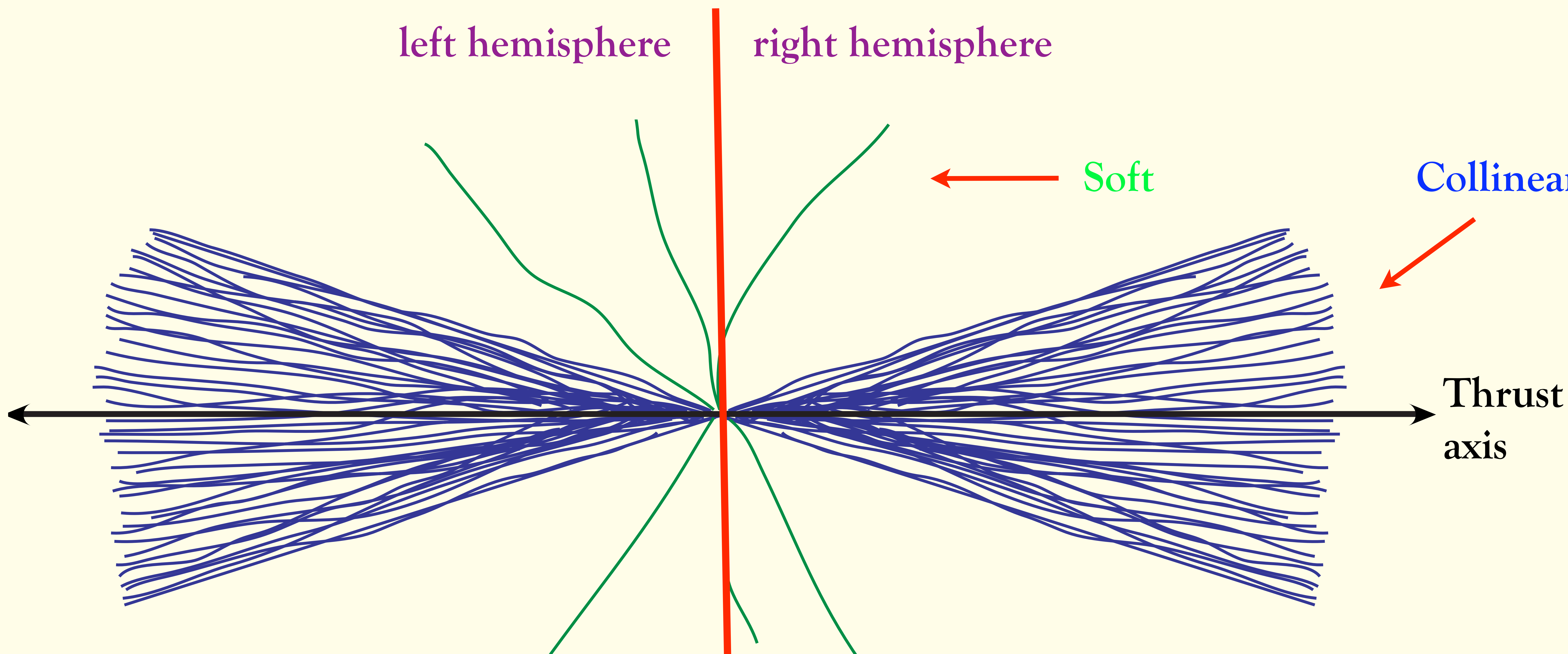
$$\times \int d\omega \, d\bar{\omega} \, |C(\omega,\bar{\omega})|^2 \langle 0|\hat{\bar{n}\!\!\!/}\chi_n|X_n\rangle\langle X_n|\overline{\chi}_{n,\omega}|0\rangle\langle 0|\overline{\chi}_{\bar{n}}|X_{\bar{n}}\rangle\langle X_{\bar{n}}|\hat{n\!\!\!/}\chi_{\bar{n},\bar{\omega}}|0\rangle \, .$$

Hard Wilson coeff. Collinear: n Collinear: $\bar{n}$

- Need to be specific about jet invariant mass definitions to make restrictions over final states explicit.
- We use Hemisphere mass definition and make the invariant mass restrictions explicit.

Hemisphere Masses

- The jet masses are defined to be the mass of all particles in each hemisphere perpendicular to the thrust axis as shown below.



$$M_{\bar{t}}^2 - m^2 \sim m\Gamma \qquad\qquad M_t^2 - m^2 \sim m\Gamma$$

The Hemisphere Scenario: Jet Invariant Masses

- The total soft momentum of the final state is the sum of the soft momentum in each hemisphere

$$k_{X_s} = k_s^a + k_s^b\,, \qquad \hat{P}_a\,|X_s> = k_s^a\,|X_s>, \quad \hat{P}_b\,|X_s> = k_s^b\,|X_s>$$

Hemisphere soft momenta

Hemisphere Projection Operators

- The invariant mass of each jet is defined to be

$$M_t^2 = (P_{X_n} + k_s^a)^2\,, \quad M_{\bar{t}}^2 = (P_{X_{\bar{n}}} + k_s^b)^2$$

- Make the invariant mass restrictions explicit by inserting the identity operator

$$\begin{aligned} 1 &= \int dM_t^2\, \delta\big((p_n + k_s^a)^2 - M_t^2\big) \int dM_{\bar{t}}^2\, \delta\big((p_{\bar{n}} + k_s^b)^2 - M_{\bar{t}}^2\big) \\ &= \int dM_t^2\, \delta\big((p_n + k_s^a)^2 - m^2 - s_t\big) \int dM_{\bar{t}}^2\, \delta\big((p_{\bar{n}} + k_s^b)^2 - m^2 - s_{\bar{t}}\big) \end{aligned}$$

...SOME ALGEBRA...

SCET Cross-section

- **In the hemisphere scenario the SCET cross section takes the form:**

$$\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2} = \sigma_0\, H_Q(Q,\mu) \int_{-\infty}^{\infty} d\ell^+ d\ell^-\; J_n(s_t - Q\ell^+,\mu)\, J_{\bar n}(s_{\bar t} - Q\ell^-,\mu)\, S_{\rm hemi}(\ell^+,\ell^-,\mu)$$

Hard Wilson Coefficient

Top Jet Function

Anti-Top Jet Function

Soft Cross Talk Function

Calculable perturbative top and antitop jet functions

$$J_n(Qr_n^+ - m^2) = \frac{-1}{2\pi Q}\int d^4x\, e^{ir_n\cdot x}\,{\rm Disc}\,\langle 0|{\rm T}\{\overline{\chi}_{n,Q}(0)\bar{\not n}\chi_n(x)\}|0\rangle$$

$$J_{\bar n}(Qr_{\bar n}^- - m^2) = \frac{1}{2\pi Q}\int d^4x\, e^{ir_{\bar n}\cdot x}\,{\rm Disc}\,\langle 0|{\rm T}\{\bar\chi_{\bar n}(x)\not n\chi_{\bar n,-Q}(0)\}|0\rangle$$

Universal nonperturbative soft function

$$S_{\rm hemi}(\ell^+,\ell^-,\mu) = \frac{1}{N_c}\sum_{X_s}\delta(\ell^+ - k_s^{+a})\delta(\ell^- - k_s^{-b})\langle 0|\overline{Y}_{\bar n}\, Y_n(0)|X_s\rangle\langle X_s|Y_n^\dagger\,\overline{Y}_{\bar n}^\dagger(0)|0\rangle$$

- **The same soft function appears in massless dijets***(Korchemsky & Sterman; Bauer, Lee, Manohar, Wise)*.

Running in SCET: Top Down vs. Bottom Up

Who Wants to Run?

$$\frac{d^2\sigma}{dM_t^2\,dM_{\bar t}^2} = \sigma_0\, H_Q(Q,\mu)\int_{-\infty}^{\infty} d\ell^+ d\ell^-\; J_n(s_t - Q\ell^+,\mu)\, J_{\bar n}(s_{\bar t} - Q\ell^-,\mu)\, S_{\rm hemi}(\ell^+,\ell^-,\mu)$$

Run the Wilson Coefficient:

Top Down

Run the jet and soft functions:

Bottom Up

Top Down	Bottom Up
$\mu\frac{d}{d\mu}H_Q(Q,\mu) = \gamma_{H_Q}(Q,\mu)\, H_Q(Q,\mu)$	$\mu\frac{d}{d\mu}J_{n,\bar n}(s,\mu) = \int ds'\;\gamma_{J_{n,\bar n}}(s-s')\; J_{n,\bar n}(s',\mu)$ $\mu\frac{d}{d\mu}S(\ell^+,\ell^-,\mu) = \int d\ell'^+ d\ell'^-\;\gamma_S(\ell^+-\ell'^+,\ell^- -\ell'^-)S(\ell'^+,\ell'^-,\mu)$

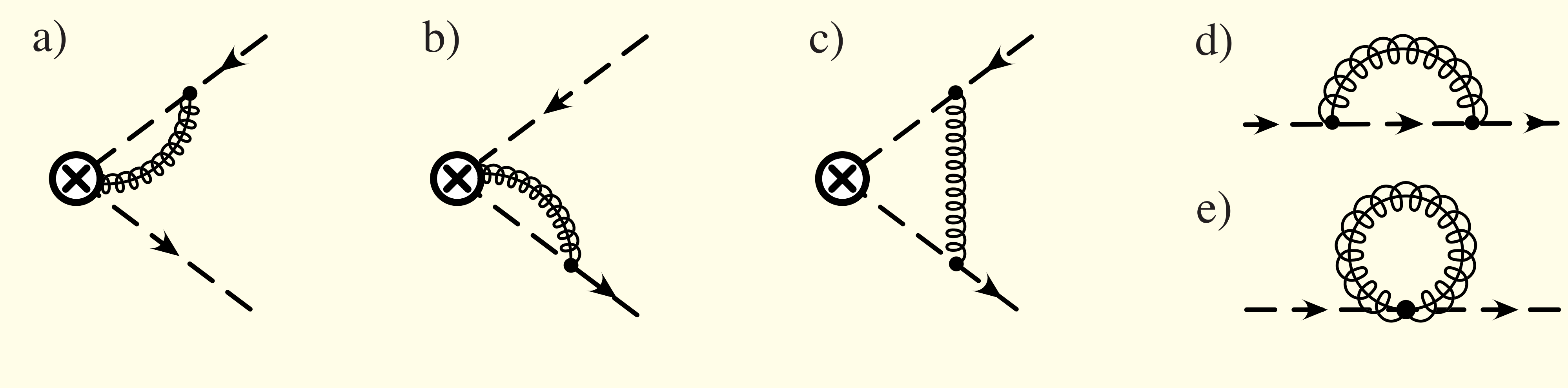
- Scale independence of the cross-section requires the equivalence of top down and bottom up running. This provides a check on the consistency of the jet invariant mass definition.

Anomalous Dimensions

Top Down:

$$\gamma_c(\mu) \;=\; -Z_c^{-1}(\mu)\mu\frac{d}{d\mu}Z_c(\mu) = -\frac{\alpha_s C_F}{\pi}\left[\ln\frac{\mu^2}{-Q^2-i\epsilon}+\frac{3}{2}\right]$$

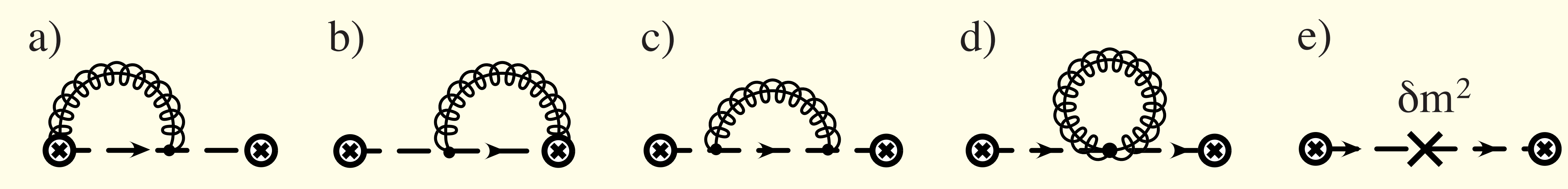
$$\gamma_H(\mu) \;=\; \gamma_c(\mu)+\gamma_c^*(\mu) = -\frac{\alpha_s C_F}{\pi}\left[2\ln\frac{\mu^2}{Q^2}+3\right]$$



One Loop Graphs: SCET Current

Bottom Up:

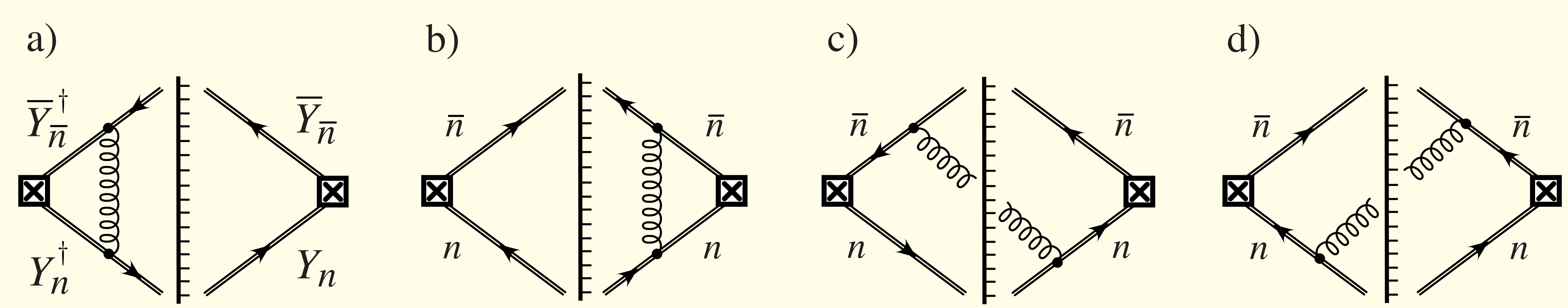
$$\gamma_{J_n}(s-s') = \frac{\alpha_s C_F}{\pi}\left\{\frac{2}{\kappa_1^2}\left[\frac{\kappa_1^2\theta(s-s')}{s-s'}\right]_+ + \delta(s-s')\left[-2\ln\left(\frac{\mu^2}{\kappa_1^2}\right)-\frac{3}{2}\right]\right\}$$



One Loop Graphs: Jet Function

$$\gamma_S(\ell^+,\ell^-) = \delta(\ell^-)\gamma_s(\ell^+)+\delta(\ell^+)\gamma_s(\ell^-)$$

$$\gamma_s(\ell^\pm) = \frac{2C_F\,\alpha_s}{\pi}\left\{\frac{1}{\kappa_2}\left[\frac{\kappa_2\theta(\ell^\pm)}{\ell^\pm}\right]_+ - \delta(\ell^\pm)\ln\left(\frac{\mu}{\kappa_2}\right)\right\}$$



One Loop Graphs: Soft Function

Evolution

Top Down

$$H_Q(Q,\mu) = U_{H_Q}(\mu,\mu_h)\, H_Q(Q,\mu_h)$$

Bottom Up

$$J_n(s,\mu) = \int ds'\; U_{J_n}(s-s',\mu,\mu_m)\;\, J_n(s',\mu_m)$$

$$J_{\bar n}(s,\mu) = \int d\bar s'\; U_{J_{\bar n}}(\bar s-\bar s',\mu,\mu_m)\;\, J_{\bar n}(\bar s',\mu_m)$$

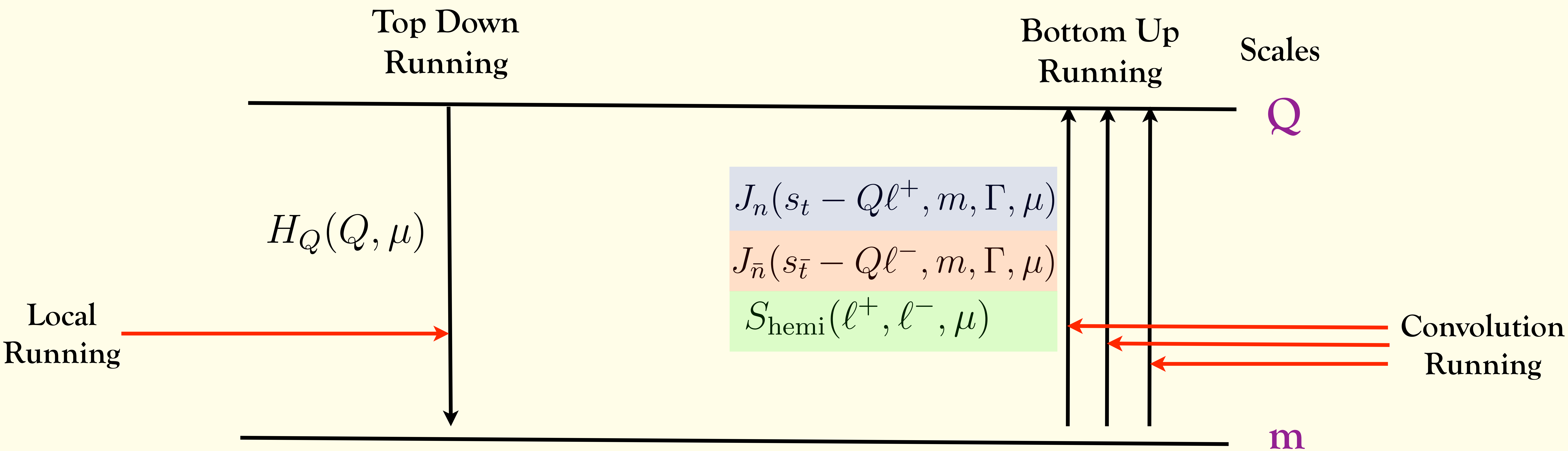
$$S_{\rm hemi}(\ell^+,\ell^-,\mu) = \int d\ell'^+ d\ell'^-\; U_S(\ell^+-\ell'^+,\ell^--\ell'^-,\mu,\mu_m)\, S_{\rm hemi}(\ell'^+,\ell'^-,\mu_m)$$

Consistency of Top Down & Bottom Up

$$U_{H_Q}(\mu,\mu_m)\,\delta(s-Q\ell'^+)\,\delta(\bar s-Q\ell'^-)$$

$$= \int d\ell^+ d\ell^-\; U_{J_n}(s-Q\ell^+,\mu,\mu_m)\, U_{J_{\bar n}}(\bar s-Q\ell^-,\mu,\mu_m)\, U_S(\ell^+-\ell'^+,\ell^--\ell'^-,\mu,\mu_m)$$

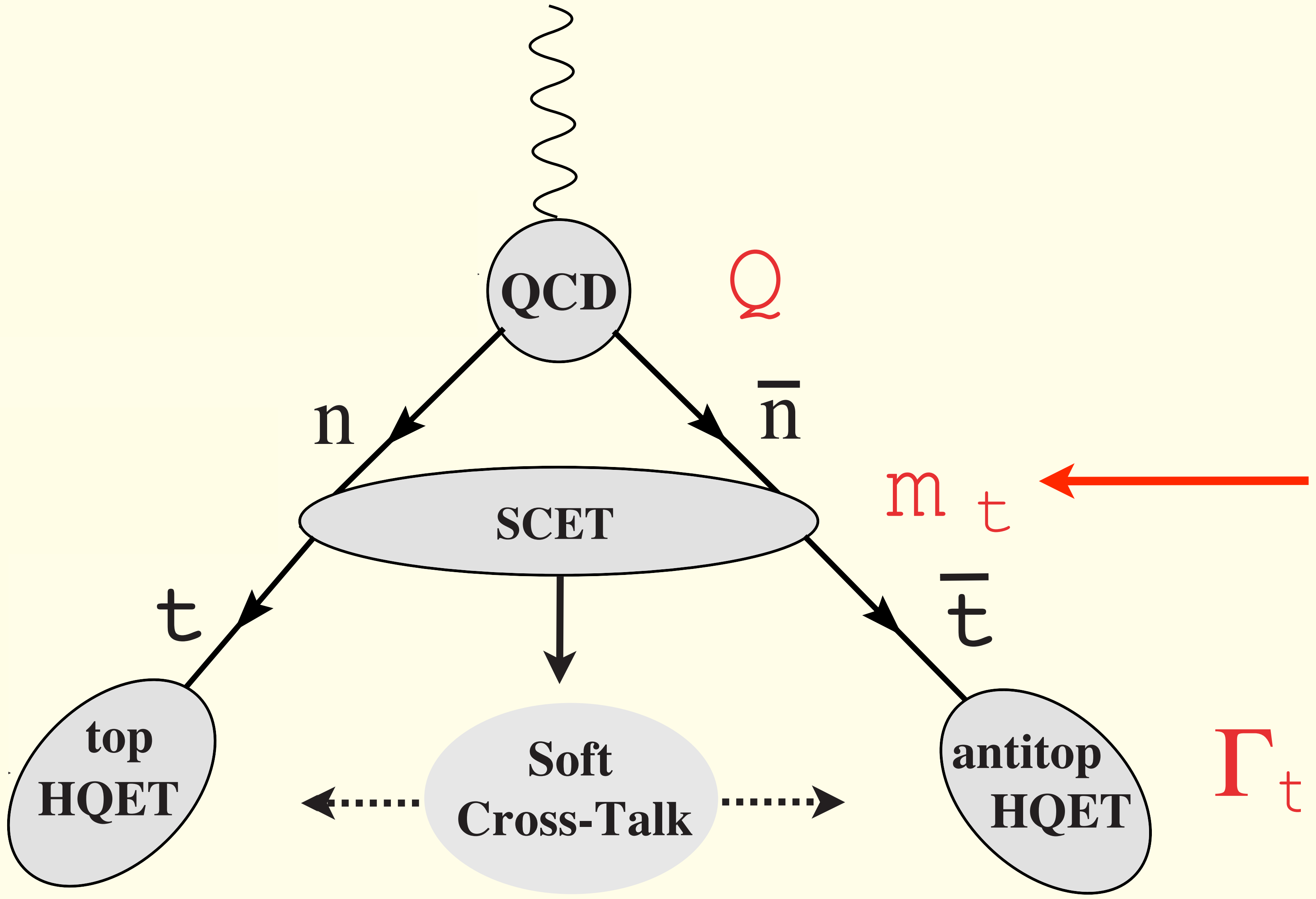
Equivalence of Top Down vs Bottom Up



- Running between Q and m is local and only affects normalization.

Matching onto HQET

- Recall the big picture:



We are here.

- Need to match SCET jet functions onto HQET and run below m.

Order m invariant mass fluctuations remain

Nonperturbative

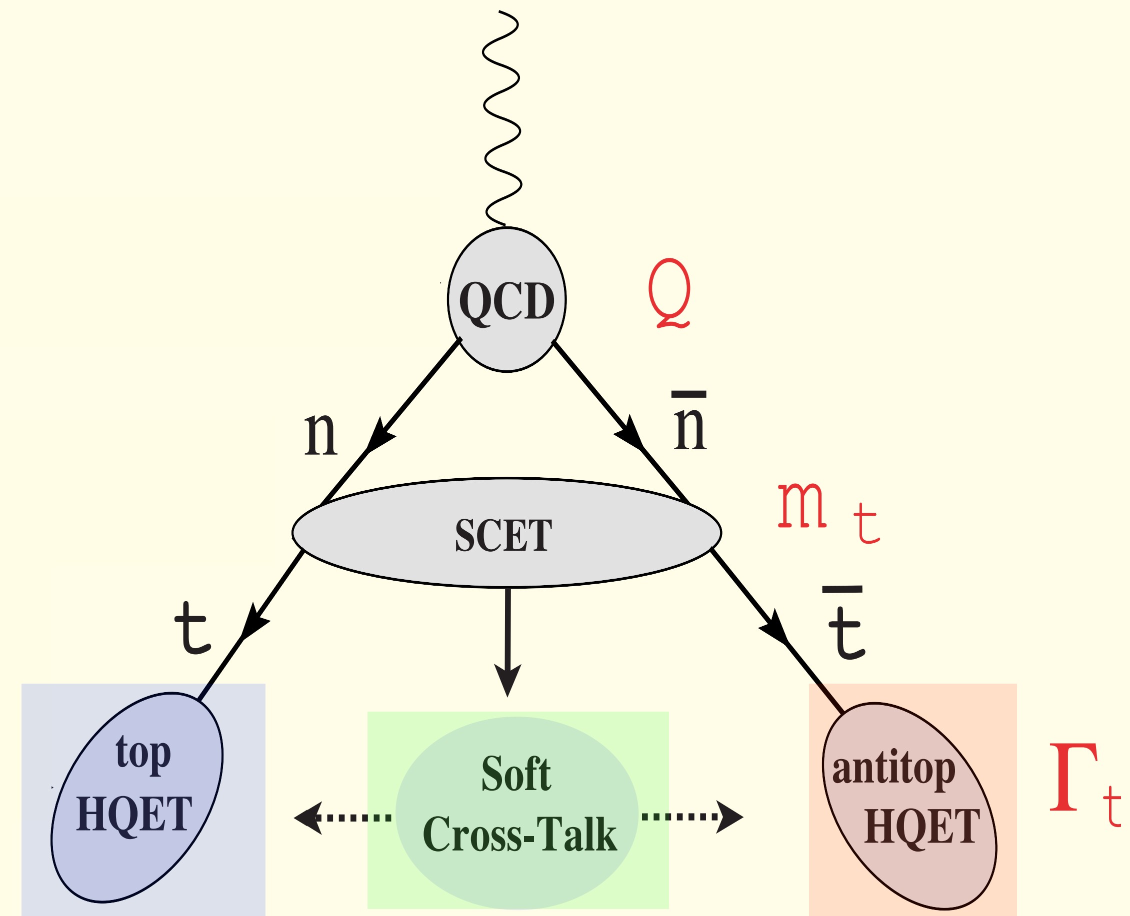
$$\left(\frac{d\sigma}{ds_t\, ds_{\bar t}}\right)_{hemi} = \sigma_0\, H_Q(Q,\mu) \int d\ell^+ d\ell^- J_n(s_t - Q\ell^+, m, \Gamma, \mu)\, J_{\bar n}(s_{\bar t} - Q\ell^-, m, \Gamma, \mu) S_{\text{hemi}}(\ell^+, \ell^-, \mu)$$

Match and run in HQET

Run below m

Boosted HQET

Decoupled Boosted HQET Sectors



Top HQET

$$\mathcal{L}_+ = \bar{h}_{v_+}\big(iv_+\cdot D_+ - \delta m + \frac{i}{2}\Gamma\big)h_{v_+},$$

Residual Mass

Width

Anti-Top HQET

$$\mathcal{L}_- = \bar{h}_{v_-}\big(iv_-\cdot D_- - \delta m + \frac{i}{2}\Gamma\big)h_{v_-}$$

Residual Mass

Width

- **Velocity labels and ultracollinear residual momenta:**

$$v_+^\mu = \left(\frac{m}{Q}, \frac{Q}{m}, \mathbf{0}_\perp\right), \qquad k_+^\mu \sim \Gamma\left(\frac{m}{Q}, \frac{Q}{m}, 1\right),$$

$$v_-^\mu = \left(\frac{Q}{m}, \frac{m}{Q}, \mathbf{0}_\perp\right), \qquad k_-^\mu \sim \Gamma\left(\frac{Q}{m}, \frac{m}{Q}, 1\right).$$

The SCET and BHQET Jet Functions

- **The SCET jet functions are given by:**

$$J_n(Qr_n^+ - m^2) = \frac{-1}{2\pi Q}\int d^4x\, e^{ir_n\cdot x}\,\mathrm{Disc}\,\langle 0|\mathrm{T}\{\overline{\chi}_{n,Q}(0)\bar{\not{n}}\chi_n(x)\}|0\rangle\,,$$

$$J_{\bar{n}}(Qr_{\bar{n}}^- - m^2) = \frac{1}{2\pi Q}\int d^4x\, e^{ir_{\bar{n}}\cdot x}\,\mathrm{Disc}\,\langle 0|\mathrm{T}\{\bar{\chi}_{\bar{n}}(x)\not{n}\chi_{\bar{n},-Q}(0)\}|0\rangle\,.$$

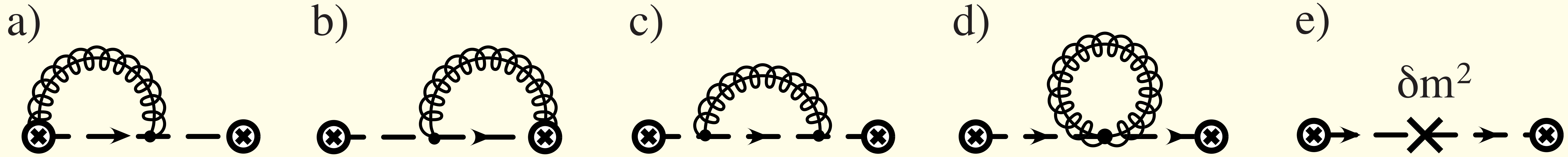
- **The BHQET jet function are given by:**

$$B_+(2v_+\cdot k) = \frac{-1}{8\pi N_c m}\int d^4x\, e^{ik\cdot x}\,\mathrm{Disc}\,\langle 0|\mathrm{T}\{\bar{h}_{v_+}(0)W_n(0)W_n^\dagger(x)h_{v_+}(x)\}|0\rangle\,,$$

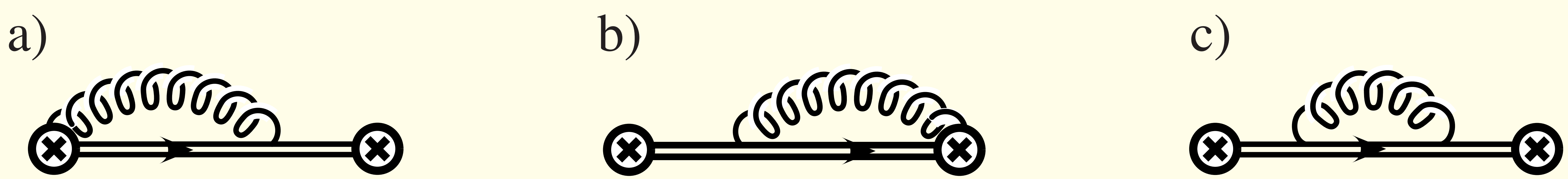
$$B_-(2v_-\cdot k) = \frac{1}{8\pi N_c m}\int d^4x\, e^{ik\cdot x}\,\mathrm{Disc}\,\langle 0|\mathrm{T}\{\bar{h}_{v_-}(x)W_{\bar{n}}(x)W_{\bar{n}}^\dagger(0)h_{v_-}(0)\}|0\rangle.$$

One Loop Matching of SCET onto BHQET

SCET



BHQET



- **Matching SCET jet functions onto bHQET:**

$$J_n(m\hat{s},\Gamma,\mu_m) = T_+(m,\mu_m)\, B_+(\hat{s},\Gamma,\mu_m)\,,$$
$$J_{\bar{n}}(m\hat{s},\Gamma,\mu_m) = T_-(m,\mu_m)\, B_-(\hat{s},\Gamma,\mu_m)$$

$$T_\pm(\mu,m) = 1 + \frac{\alpha_s C_F}{4\pi}\left(\ln^2\frac{m^2}{\mu^2} - \ln\frac{m^2}{\mu^2} + 4 + \frac{\pi^2}{6}\right).$$

$$H_m\Big(m,\mu_m\Big) = T_+(m,\mu_m)T_-(m,\mu_m)$$

- **Note that the logs in the Wilson coefficient vanish by choosing scale:**

$$\mu = m$$

Final Form of Differential Cross-Section

Hard Production modes integrated out

"Hard" collinear gluons integrated out

Evolution and decay of top quark close to mass shell

Non-perturbative Cross talk

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar{t}}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right)$$

$$\times \int_{-\infty}^{\infty} d\ell^+ d\ell^- \, B_+\left(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat{s}_{\bar{t}} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

Running in bHQET: Top Down vs Bottom Up

Who Wants to Run?

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar{t}}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big)$$

Run the Wilson Coefficient

$$\times \int_{-\infty}^{\infty} d\ell^+ d\ell^- \; B_+\Big(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\, B_-\Big(\hat{s}_{\bar{t}} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\, S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

Run the Jet & Soft functions

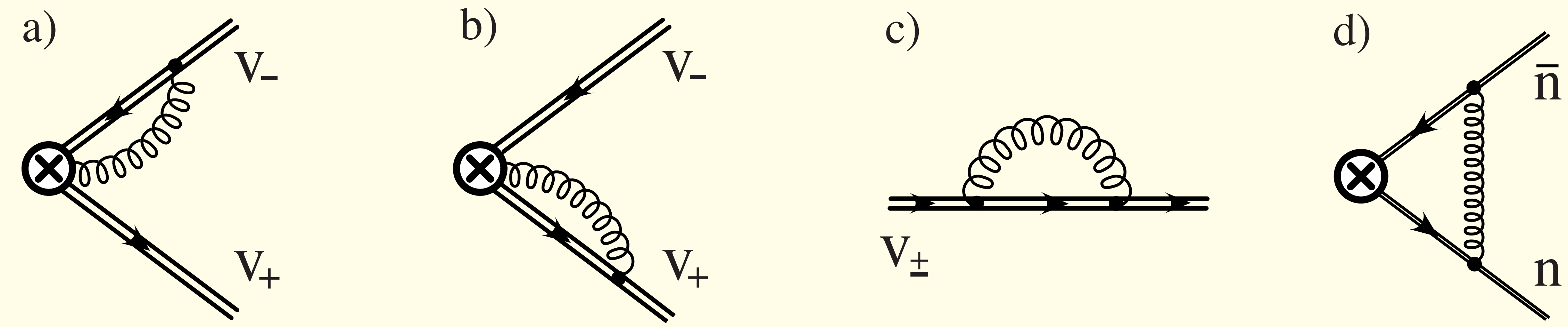
Top Down	Bottom Up
$\mu \frac{d}{d\mu} H_m\Big(m, \frac{Q}{m}, \mu\Big) = \gamma_{H_m}\Big(\frac{Q}{m}, \mu\Big)\, H_m\Big(m, \frac{Q}{m}, \mu\Big)$	$\mu \frac{d}{d\mu} B_\pm(\hat{s}, \mu) = \int d\hat{s}'\, \gamma_{B_\pm}(\hat{s} - \hat{s}')\, B_\pm(\hat{s}', \mu)$ $\mu \frac{d}{d\mu} S_{hemi}(\ell^+, \ell^-, \mu) = \int d\ell'^+ d\ell'^-\, \gamma_S(\ell^+ - \ell'^+, \ell^- - \ell'^-) S_{hemi}(\ell'^+, \ell'^-, \mu)$

Anomalous Dimensions

Top Down:

$$\gamma_{C_m}(\mu) = Z_{C_m}^{-1}(\mu)\,\mu\frac{d}{d\mu}Z_{C_m}(\mu) = -\frac{\alpha_s C_F}{\pi}\left[\ln\frac{-Q^2-i0}{m^2}-1\right]$$

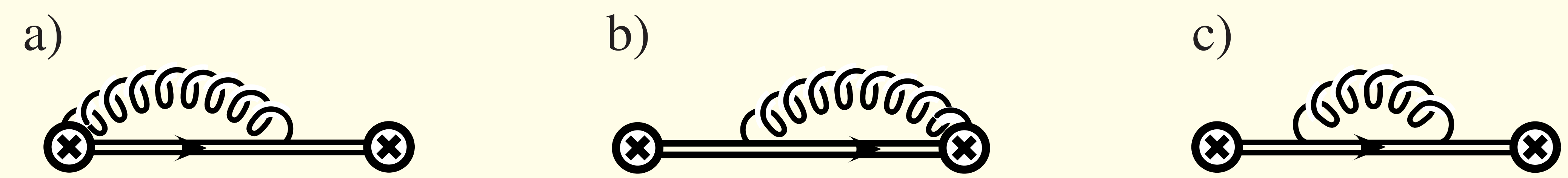
$$\gamma_{H_m}(\mu) = \gamma_{C_m}(\mu)+\gamma_{C_m}(\mu)^* = -\frac{\alpha_s C_F}{\pi}\left[2\ln\frac{Q^2}{m^2}-2\right].$$



One Loop Graphs: bHQET Current

Bottom Up:

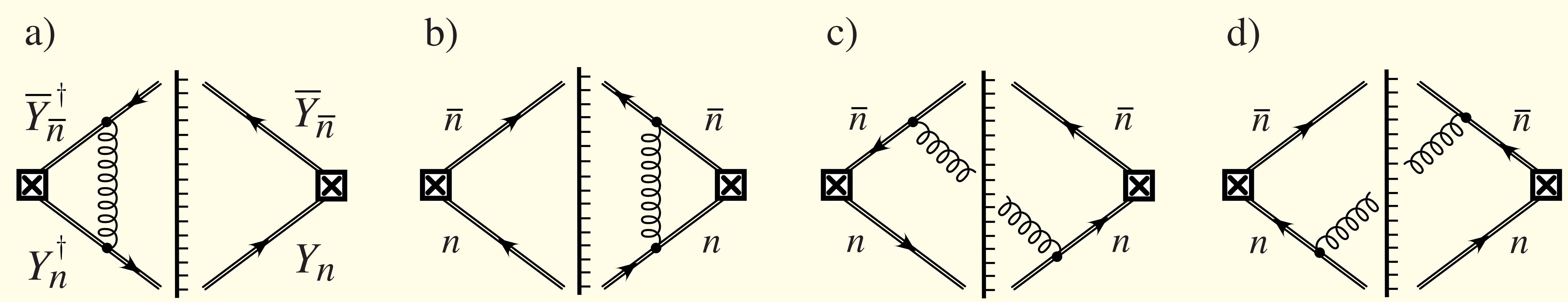
$$\gamma_{B_\pm}(\hat{s}-\hat{s}',\mu) = \frac{\alpha_s C_F}{\pi}\left\{2\left[\frac{\kappa_3\theta(\hat{s}'-\hat{s})}{\hat{s}'-\hat{s}}\right]_+ - \left[2\ln\left(\frac{\mu}{\kappa_3}\right)+1\right]\delta(s'-s)\right\}$$



One Loop Graphs: bHQET Jet Function

$$\gamma_S(\ell^+,\ell^-) = \delta(\ell^-)\gamma_s(\ell^+)+\delta(\ell^+)\gamma_s(\ell^-)$$

$$\gamma_s(\ell^\pm) = \frac{2C_F\,\alpha_s}{\pi}\left\{\frac{1}{\kappa_2}\left[\frac{\kappa_2\theta(\ell^\pm)}{\ell^\pm}\right]_+ - \delta(\ell^\pm)\ln\left(\frac{\mu}{\kappa_2}\right)\right\}$$



One Loop Graphs: Soft Function

Evolution

Top Down

$$H_m(\mu) = U_{H_m}(\mu, \mu_m) H_m(\mu_m)$$

Bottom Up

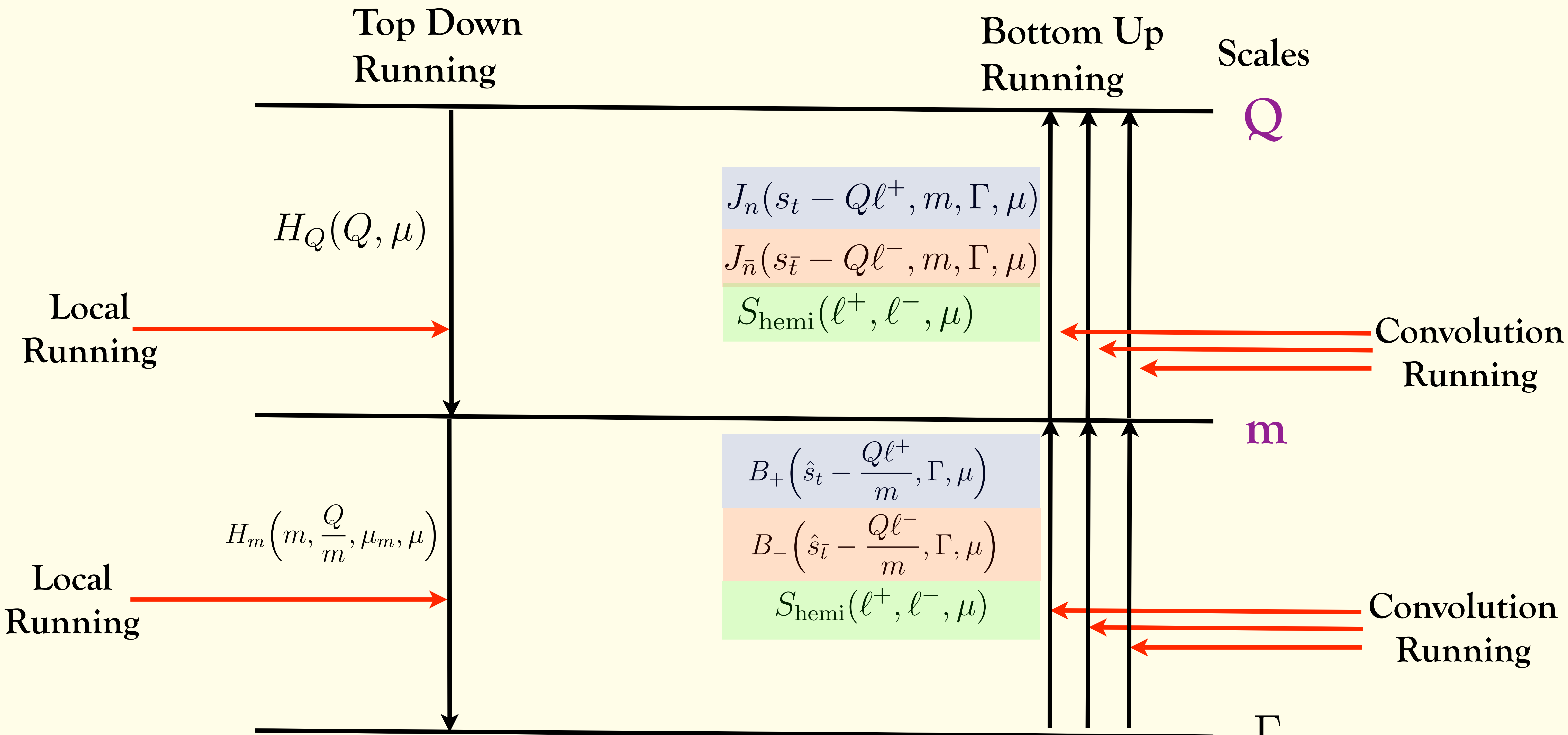
$$B_\pm(\hat{s}, \mu) = \int d\hat{s}' \; U_{B_\pm}(\hat{s} - \hat{s}', \mu, \mu_\Gamma) \; B_\pm(\hat{s}', \mu_\Gamma)$$

$$S_{\rm hemi}(\ell^+, \ell^-, \mu) = \int d\ell'^+ d\ell'^- \; U_S(\ell^+ - \ell'^+, \ell^- - \ell'^-, \mu, \mu_m) \; S_{\rm hemi}(\ell'^+, \ell'^-, \mu_m)$$

Consistency of top down & bottom up

$$U_{H_m}(\mu, \mu_\Delta) \, \delta\Big(\hat{s} - \frac{Q\ell'^+}{m}\Big) \, \delta\Big(\hat{\bar{s}} - \frac{Q\ell'^-}{m}\Big) = \int d\ell^+ d\ell^- \; U_{B_+}\Big(\hat{s} - \frac{Q\ell^+}{m}, \mu, \mu_\Delta\Big) U_{B_-}\Big(\bar{s} - \frac{Q\ell^-}{m}, \mu, \mu_\Delta\Big) U_S(\ell^+ - \ell'^+, \ell^- - \ell'^-, \mu, \mu_\Delta)$$

Equivalence of Top-Down vs. Bottom Up



- Running between the different scales only affects only the normalization!

Short Distance Mass for Jets

Connecting the Observable to a Short Distance Mass Scheme

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\, S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

- **We have an analytic formula for the double differential jet invariant mass distribution in terms of the pole mass.**

- **We can now switch to a short distance mass scheme in bHQET.**

$$m_{\rm pole} = m + \delta m$$

Switching Mass Schemes in bHQET

Top HQET

$$\mathcal{L}_+ = \bar{h}_{v_+}\big(iv_+ \cdot D_+ - \delta m + \frac{i}{2}\Gamma\big)h_{v_+},$$

Anti-Top HQET

$$\mathcal{L}_- = \bar{h}_{v_-}\big(iv_- \cdot D_- - \delta m + \frac{i}{2}\Gamma\big)h_{v_-}$$

- Power counting in bHQET requires

$$\delta m \sim \hat{s}_t \sim \hat{s}_{\bar{t}} \sim \Gamma$$

- Note that this power counting breaks down in the $\overline{\mathrm{MS}}$ scheme:

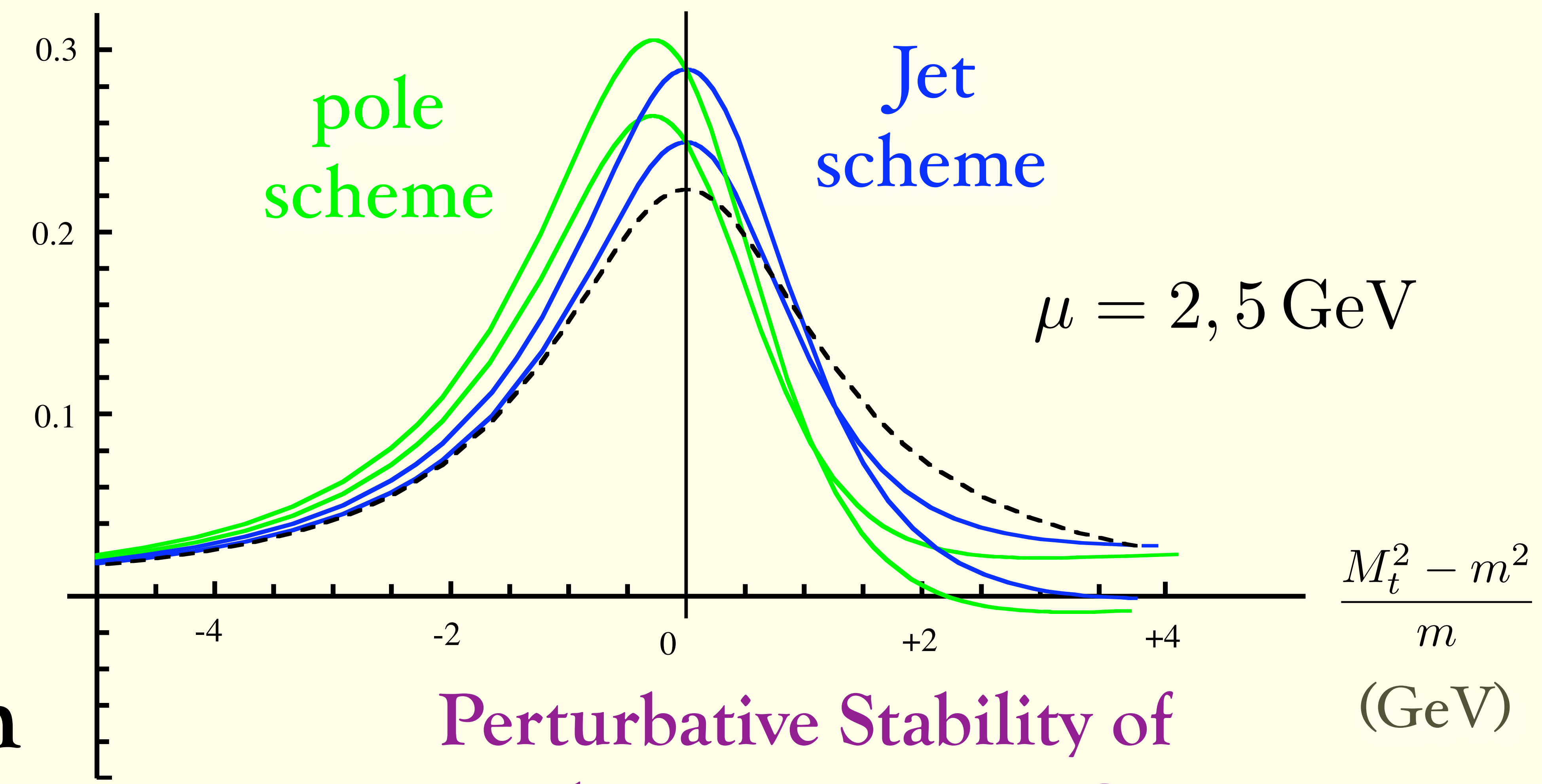
$$\delta\overline{m} \sim \alpha_s \overline{m} \gg \Gamma$$

- We need a short distance mass that respects the power counting of bHQET.

Short Distance Top Jet Mass

- **Define the short distance top jet mass scheme as:**

$$\left.\frac{dB_+(\hat{s},\mu,\delta m_J)}{d\hat{s}}\right|_{\hat{s}=0} = 0$$



Perturbative Stability of Peak Position at NLO.

- **In the jet mass scheme the NLO jet function is modified as:**

$$\tilde{B}_\pm(\hat{s},\mu) \;=\; B_\pm(\hat{s},\mu) + \frac{1}{\pi m_J}\,\frac{(4\,\hat{s}\,\Gamma)\,\delta m_J}{(\hat{s}^2+\Gamma^2)^2}$$

- **At NLO the jet mass is related to the pole mass scheme as follows:**

$$m_J(\mu) = m_{\rm pole} - \Gamma\frac{\alpha_s(\mu)}{3}\Big[\ln\Big(\frac{\mu}{\Gamma}\Big) + \frac{3}{2}\Big]$$

Invariant Mass Distribution: Analysis

Jet and Soft Functions

- Jet functions are Breit Wigner distributions at tree level:

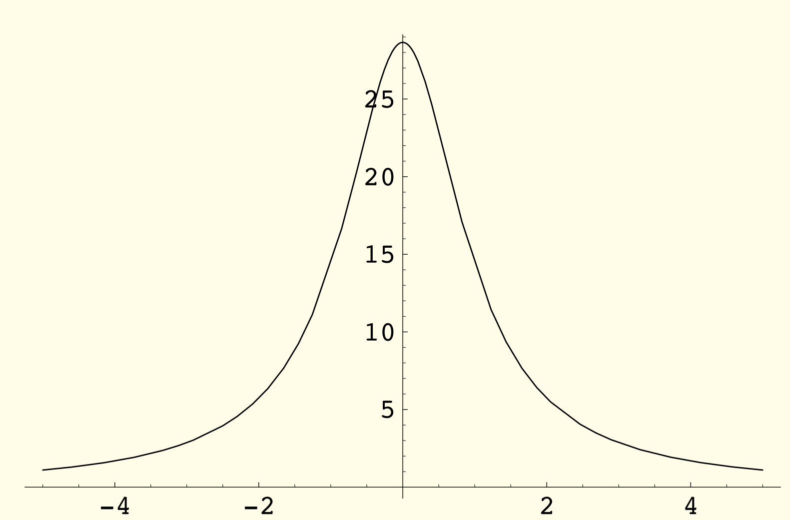
$$B_\pm^{\rm tree}(\hat{s},\Gamma) = \frac{1}{\pi m}\,\frac{\Gamma}{\hat{s}^2+\Gamma^2}$$

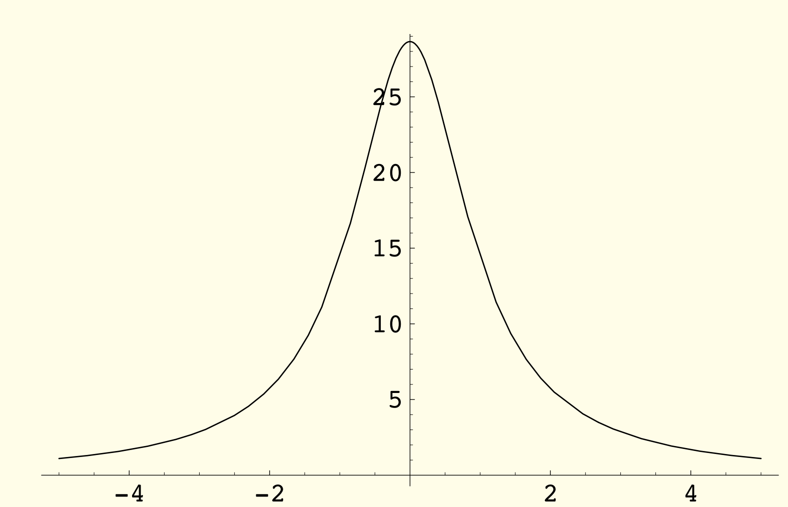
- Use shape function extracted from massless dijets *(Korchemsky & Sterman)*:

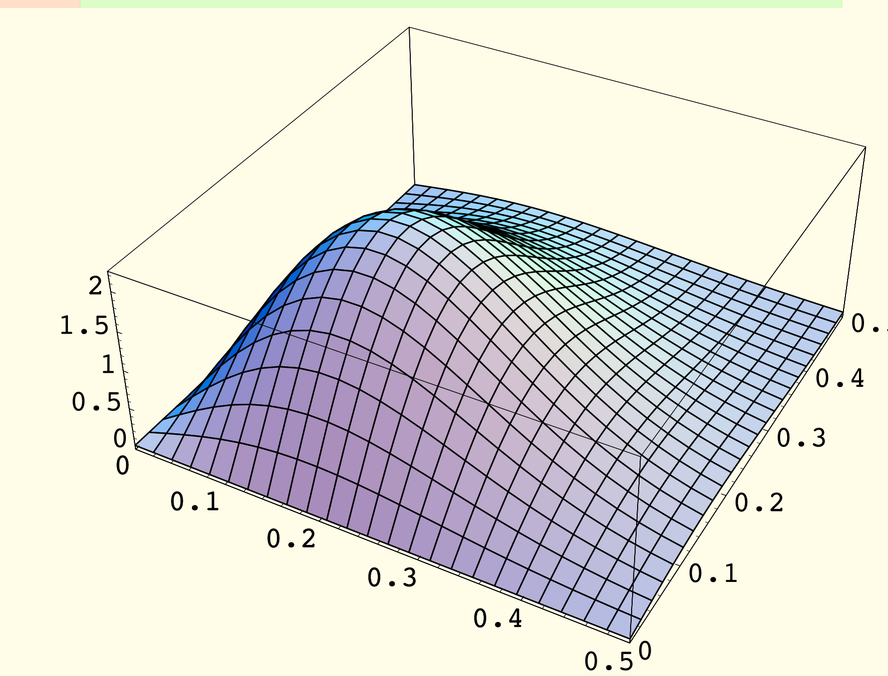
$$S^{\rm M1}_{\rm hemi}(\ell^+,\ell^-) = \theta(\ell^+)\theta(\ell^-)\frac{\mathcal{N}(a,b)}{\Lambda^2}\Big(\frac{\ell^+\ell^-}{\Lambda^2}\Big)^{a-1}\exp\Big(\frac{-(\ell^+)^2-(\ell^-)^2-2b\ell^+\ell^-}{\Lambda^2}\Big)$$

$$a=2\,,\qquad b=-0.4\,,\qquad \Lambda = 0.55\,\mathrm{GeV}$$

$$\Big(\frac{d^2\sigma}{dM_t^2\,dM_{\bar t}^2}\Big)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m,\frac{Q}{m},\mu_m,\mu\Big) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat{s}_t - \frac{Q\ell^+}{m},\Gamma,\mu\Big)\, B_-\Big(\hat{s}_{\bar t} - \frac{Q\ell^-}{m},\Gamma,\mu\Big)\, S_{\rm hemi}(\ell^+,\ell^-,\mu)$$







Tree level BWs

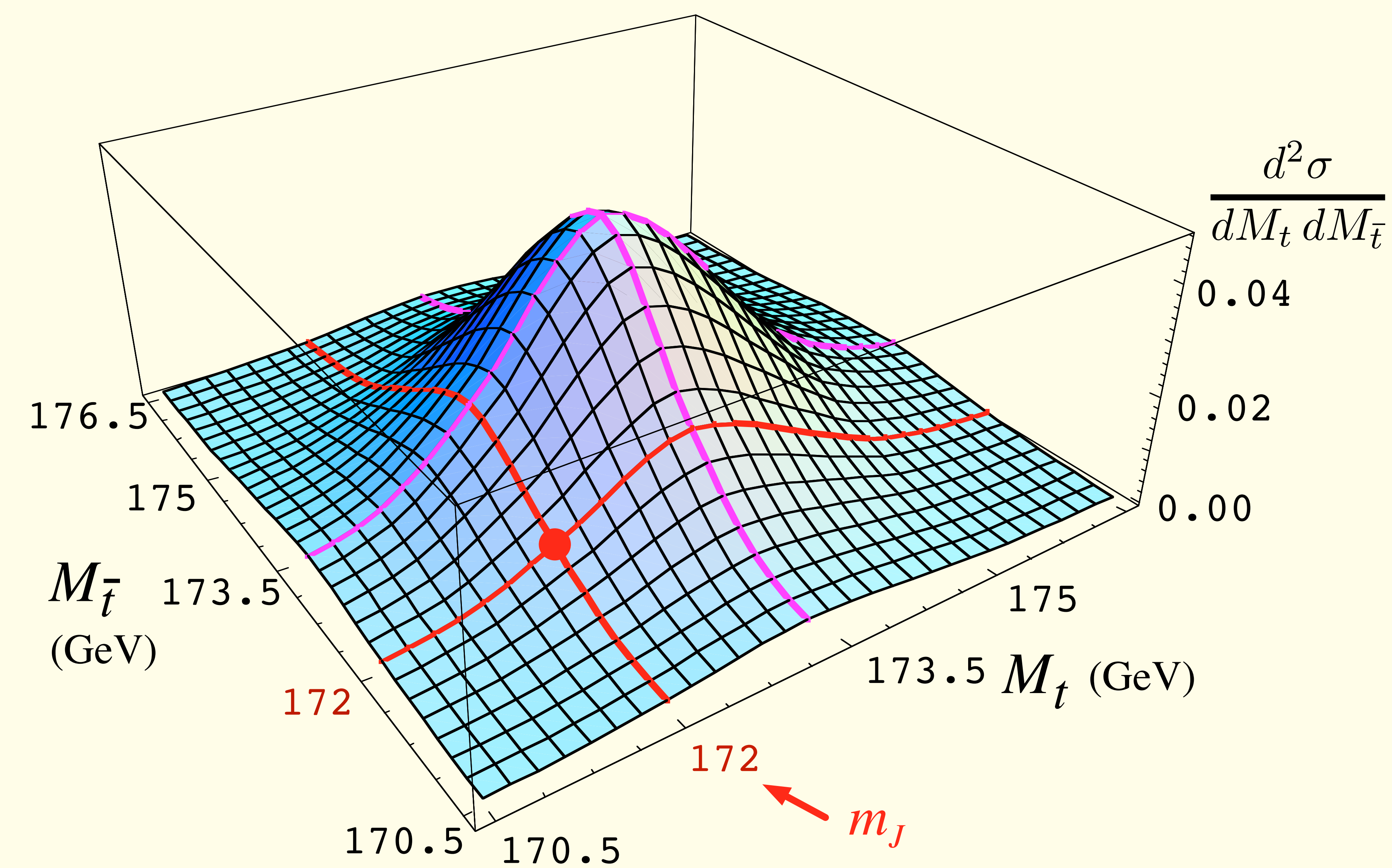
Shape function

Double Differential Invariant Mass Distribution

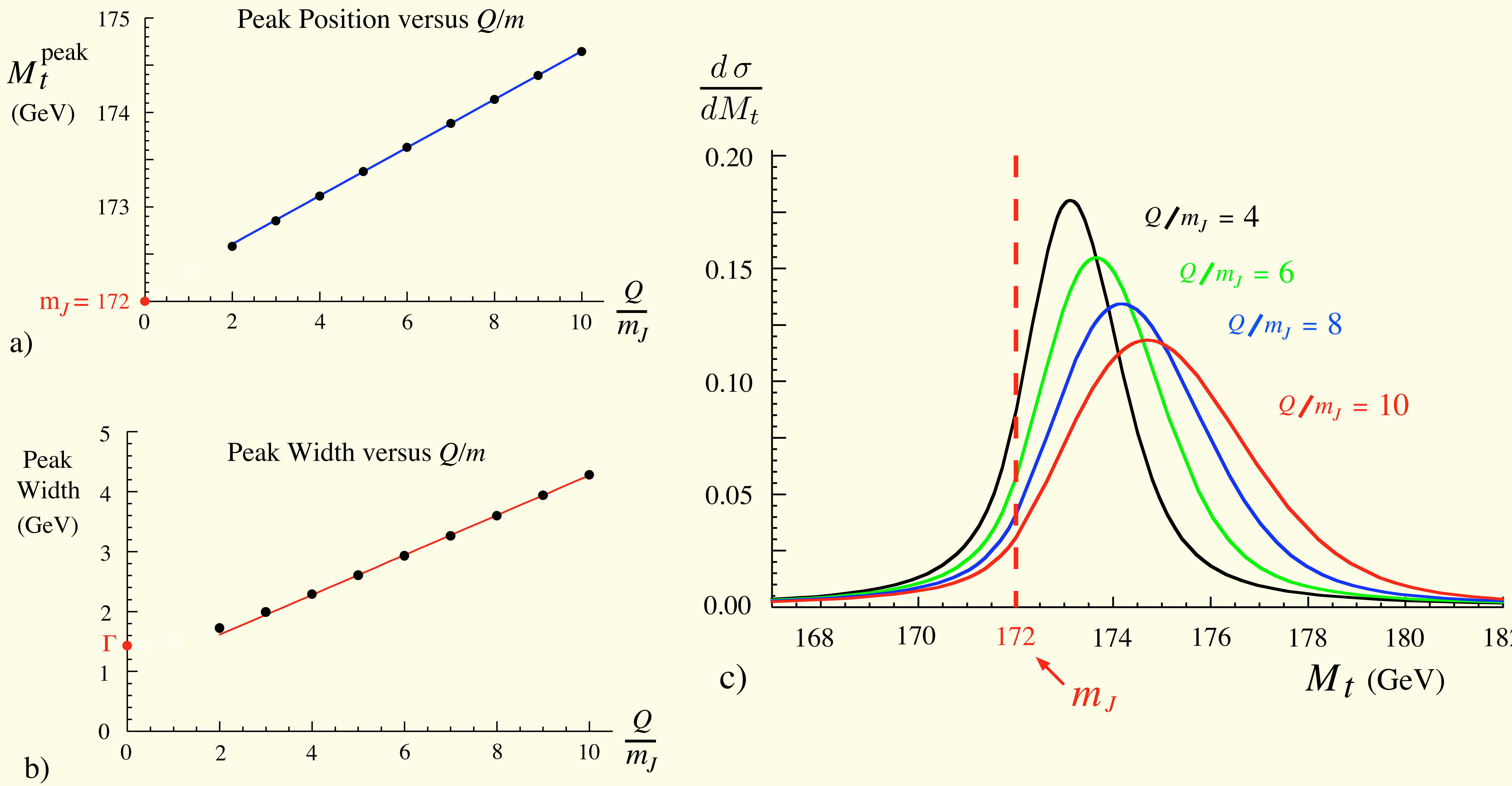
$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\text{hemi}} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big)$$
$$\times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\, S_{\text{hemi}}(\ell^+, \ell^-, \mu)$$

- **Measured peak position is shifted away from the short distance mass value due to the nonperturbative soft function.**

- **Naive Breit Wigner fit not valid even at tree level.**



NonPerturbative Effects in Single Differential Distribution



- Peak position shifts linearly with the center of mass energy.
- Width of distribution also shifts linearly with center of mass energy.

Other Event Shapes

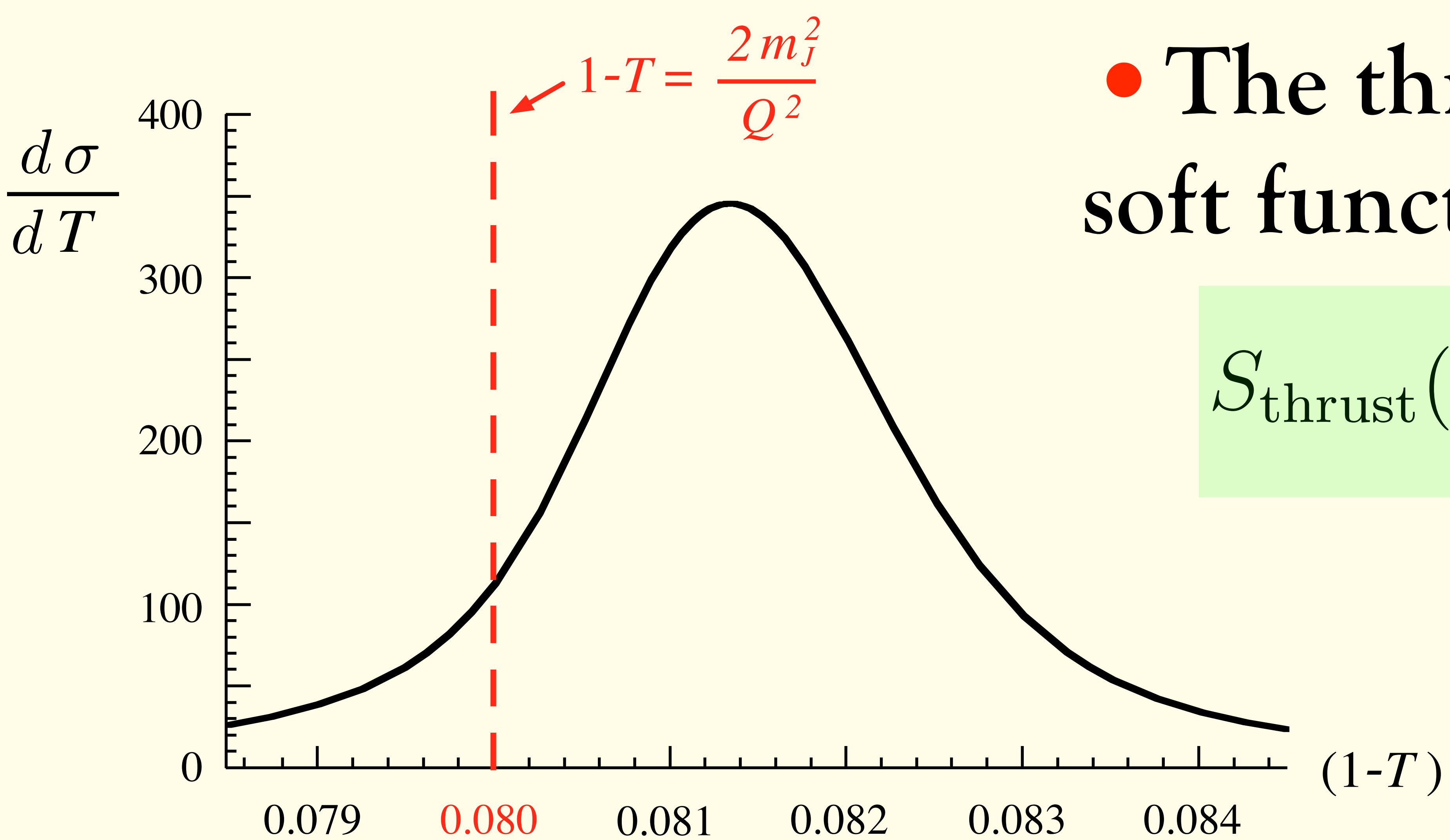
Thrust Distribution

- The thrust variable is related to the jet invariant masses as:

$$1 - T = (M_t^2 + M_{\bar{t}}^2)/Q^2$$

- Using the above relation one can obtain the thrust distribution:

$$\frac{d\sigma}{dT} = \sigma_0^H(\mu) \int_{-\infty}^{\infty} ds_t \, ds_{\bar{t}} \; \tilde{B}_+\Big(\frac{s_t}{m_J}, \Gamma, \mu\Big) \, \tilde{B}_-\Big(\frac{s_{\bar{t}}}{m_J}, \Gamma, \mu\Big) \, S_{\rm thrust}\Big(1 - T - \frac{(2m_J^2 + s_t + s_{\bar{t}})}{Q^2}, \mu\Big)$$



Thrust Distribution

- The thrust soft function is related to the hemisphere soft function as:

$$S_{\rm thrust}(\tau, \mu) = \int_0^{\infty} d\ell^+ \, d\ell^- \delta\Big(\tau - \frac{(\ell^+ + \ell^-)}{Q}\Big) S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

Conclusions

- We have developed an analytic framework that gives a clear and well defined relation between the short distance top mass and reconstruction from jets:
    - We define a new short distance mass suitable for reconstruction from jets.
    - Peak position is shifted away from the short distance top mass value by universal nonperturbative effects.
    - The shift is linear in the center of mass energy.
    - The width of the distribution also grows linearly with energy.
    - Large logarithms only affect the overall normalization of the distribution.

- EFT approach allows for factorization, power corrections, resummation, and universal characterization of non-perturbative effects.

- One can generalize this approach for different jet algorithms especially those suited for the LHC and work is in progress.