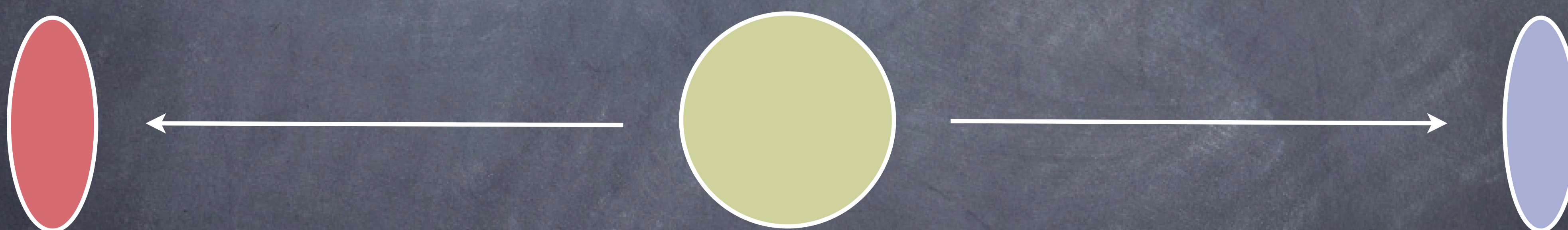
A new reality:
$B \to \pi\pi$ annihilation in SCET



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A,L,R,S hep-ph/0607001
A,R,S hep-ph/0611356

Outline

- motivation
- classification of power-suppressed contributions
- local annihilation
- complex annihilation
- three-parton annihilation

$$M_1 \longleftarrow B \longrightarrow M_2$$

with $M_1\, M_2 \in \{\pi, \rho, K, K^*\}$

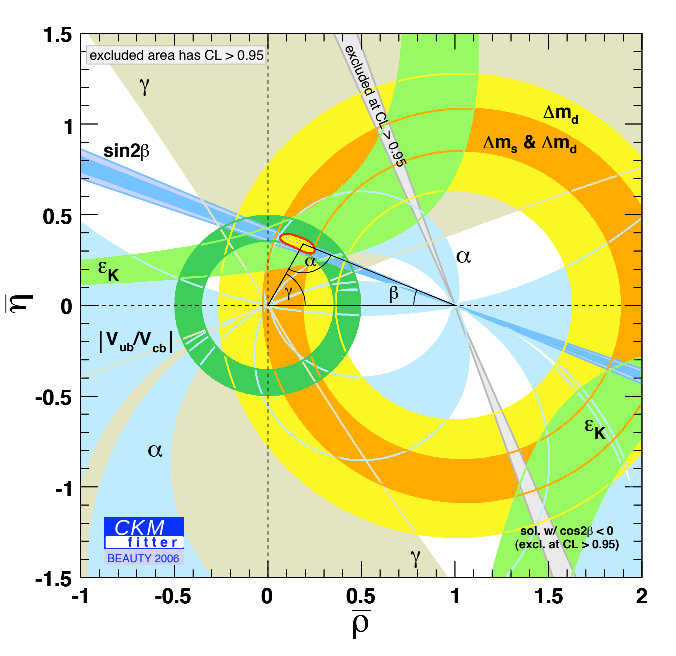
- probe strong dynamics and $\cancel{CP}$
- B-factories: $\overline{\mathrm{Br}}, A, C, S$

$$A_{K\pi} = -0.107 \pm 0.018$$

$B \to M_1 M_2$ in the Standard Model

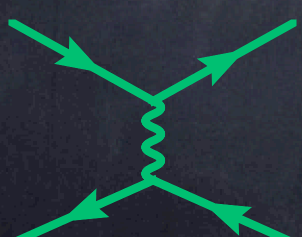
$$\begin{array}{r} V_{ub}V_{uf}^* \\ V_{cb}V_{cf}^* \\ +V_{tb}V_{tf}^* \\ \hline 0 \end{array}$$

$$\xrightarrow{f=d}$$



$$A(\bar{B} \to M_1 M_2) =$$

$$V_{ub}V_{uf}^* \, T + V_{cb}V_{cf}^* \, P$$



"tree"

"penguin"

QCD for P,T:
expand in small #'s

- SU(2) isospin $m_{u,d}/\Lambda_{\rm QCD} \approx 0.03$
- power counting $\Lambda_{\rm QCD}/E_M \approx 0.2$
- strong coupling $\alpha_s \approx 0.1 - 0.35$

reduces # of non-perturbative inputs

A little theory + a lotta experiment:

(Jain, Rothstein, Stewart 07)

$$10^3\,\hat{P}^{\pi\pi} = (1.82 \pm 0.86) - i(2.99 \pm 0.74)$$

$$10^3\,\hat{P}^{K\pi} = -(4.45 \pm 0.81) + i(3.23 \pm 1.11)$$

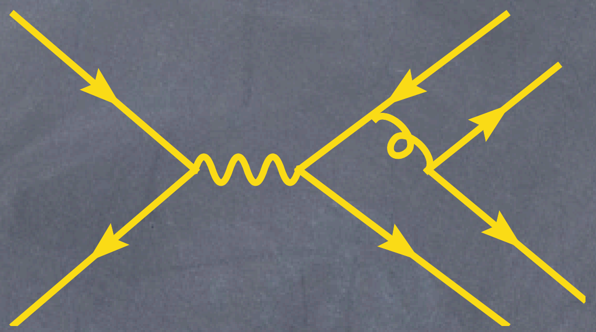
$$\hat{} \equiv / \frac{G_F m_B^2 \mathrm{GeV}}{\sqrt{2}}$$

large imaginary parts

$$T = T^* > 0$$

Chasing large imaginary penguins

- ☒ ¿annihilation?

  PQCD (Keum, Li, Sanda): yes

  QCDF (Beneke, Buchalla, Neubert, Sachrajda): not computable

  SCET (CA, Ligeti, Rothstein, Stewart): no

- ☒ ¿large missing short distance phase?

  SCET (Beneke, Jager), (Jain, Rothstein, Stewart): no

- ☐ ¿ charm loops in the resonance region?

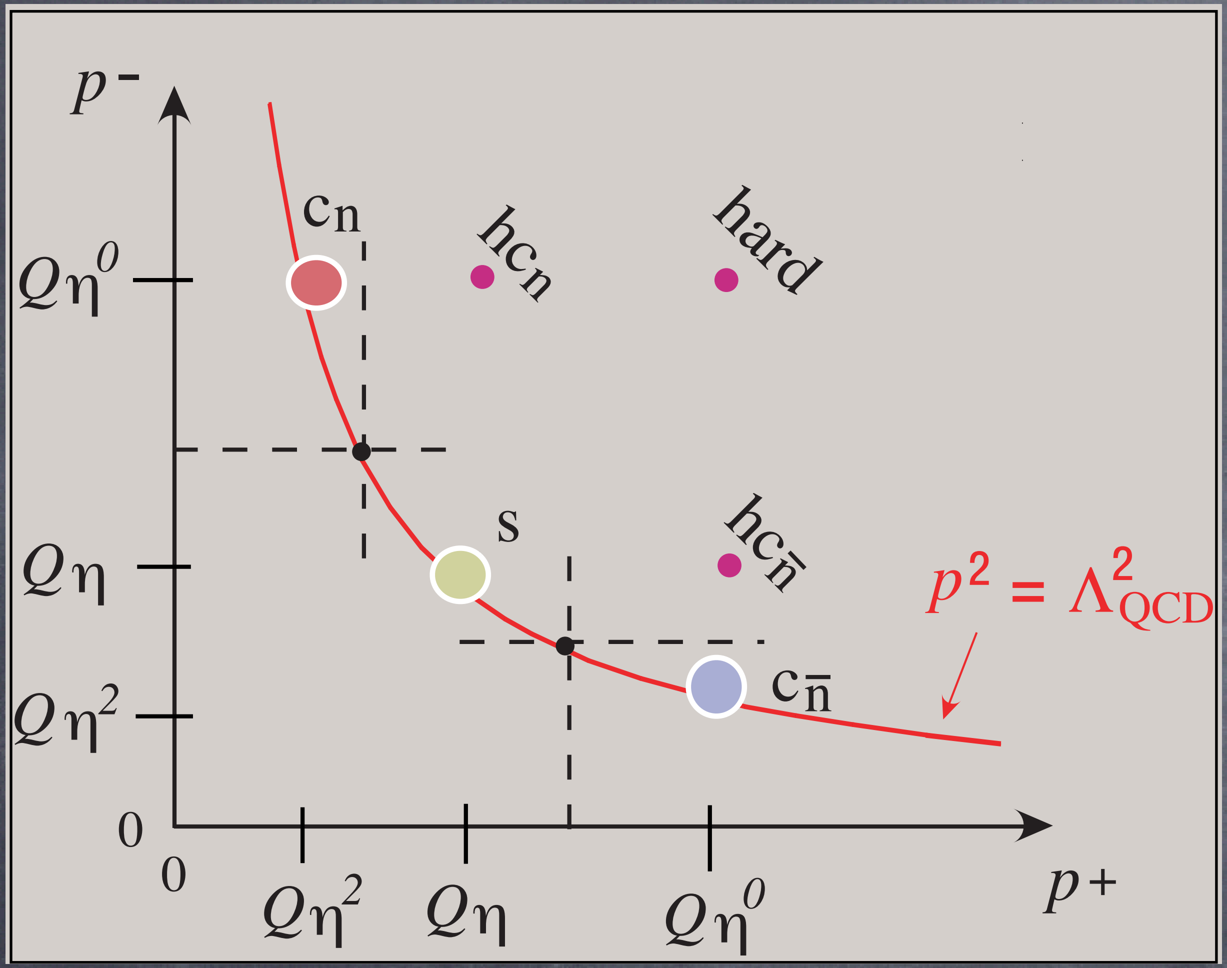
- ☐ ¿ new physics?

$B \to M_1 M_2$: the SCET calculation

$$p = \overset{p^-}{\bar{n}\cdot p}\,\frac{n}{2} + \overset{p^+}{n\cdot p}\,\frac{\bar{n}}{2} + p_\perp$$

SCET$_{\mathrm{II}}$ power expansion: series in $\eta \sim \dfrac{\Lambda_{\mathrm{QCD}}}{Q}$

SCET$_{\mathrm{II}}$ degrees of freedom



$$\zeta_p \equiv \frac{p^-}{p^+}$$

mode:	n-collinear	soft	$\bar{n}$-collinear
$(+,-,\perp)$:	$Q(\eta^2,1,\eta)$	$Q(\eta,\eta,\eta)$	$Q(1,\eta^2,\eta)$
fields:	ξ_n, A_n	h_v, q_s, A_s	$\xi_{\bar{n}}, A_{\bar{n}}$

Decay amplitude

$$H_W^{\mathrm{QCD}}$$

$$H_W^{\mathrm{I}}$$

$$A(\bar{B} \to M_1 M_2)$$

$$= -i \langle M_1 M_2 | H_W^{\mathrm{II}} | \bar{B} \rangle$$

$$= -i * \text{matching coefficients}$$

$$\otimes \langle M_1 | c_n \text{ fields} | 0 \rangle$$

$$\otimes \langle M_2 | c_{\bar{n}} \text{ fields} | 0 \rangle$$

$$\otimes \langle 0 | \text{soft fields} | \bar{B} \rangle$$

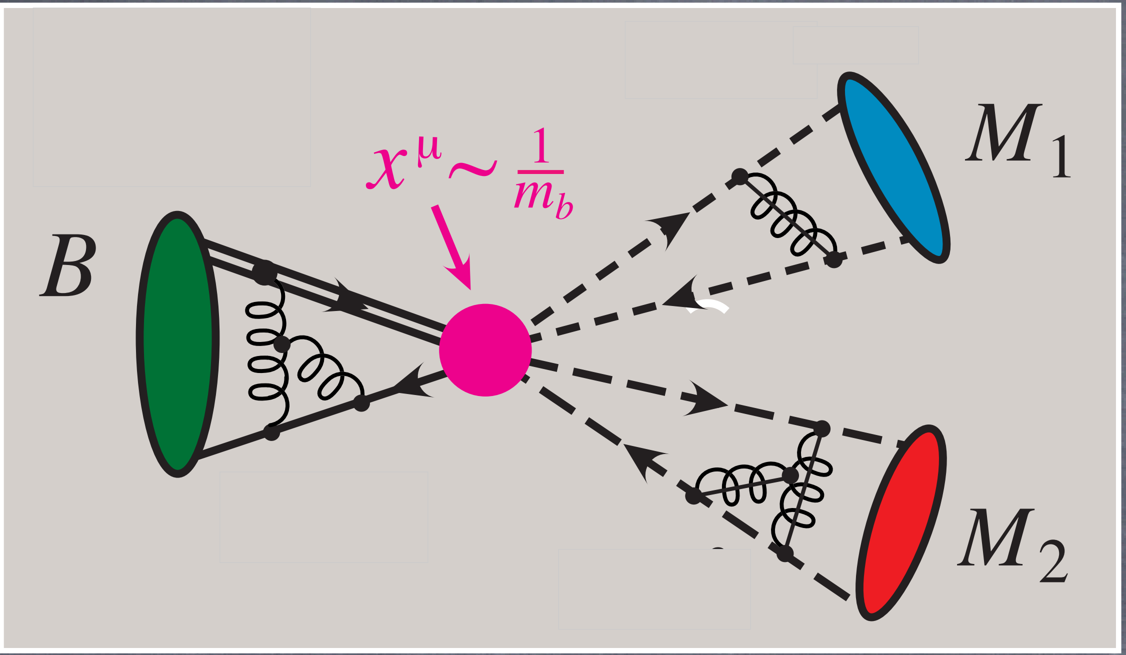
Classification of power-suppressed amplitudes

Order in Λ/m_b	Time-ordered products in $\mathrm{SCET_I}$	Perturbative order Annihilation	Other
$A^{(0)}$	$Q_i^{(0)}\mathcal{L}_{\xi q}^{(1)},\ Q_i^{(0)}\mathcal{L}_{\xi q}^{(2)},\ Q_i^{(0)}\mathcal{L}_{\xi q}^{(1)}\mathcal{L}^{(1)}$	—	$\alpha_s(\mu_i)$
	$Q_i^{(1)}\mathcal{L}_{\xi q}^{(1)}$	—	$\alpha_s(\mu_i)$
$A^{(1)}$	$Q_i^{(j'=0,1)}\mathcal{L}_{\xi q}^{(j\le 4)}\,\Pi_i\,\mathcal{L}^{(k_i)}$	—	$\alpha_s(\mu_i)$
	$Q_i^{(4)}$	$\alpha_s(\mu_h)$	—
	$Q_i^{(2)}\mathcal{L}_{\xi q}^{(1)}$	$\alpha_s(\mu_h)$	$\alpha_s(\mu_i)$
	$Q_i^{(0)}\left[\mathcal{L}_{\xi q}^{(1)}\right]^3,\ Q_i^{(0)}\left[\mathcal{L}_{\xi q}^{(1)}\right]^3\mathcal{L}^{(1)}$	$\alpha_s^2(\mu_i)/\pi$	$\alpha_s^2(\mu_i)/\pi$
	$Q_i^{(0)}\left[\mathcal{L}_{\xi q}^{(1)}\right]^2\mathcal{L}_{\xi q}^{(2)},\ Q_i^{(1)}\left[\mathcal{L}_{\xi q}^{(1)}\right]^3$	$\alpha_s^2(\mu_i)/\pi$	$\alpha_s^2(\mu_i)/\pi$
	$Q_i^{(2)}\left[\mathcal{L}_{\xi q}^{(1)}\right]^2$	—	$\alpha_s^2(\mu_i)/\pi$
	$Q_i^{(2)}\mathcal{L}_{\xi q}^{(1)}\mathcal{L}^{(1)},\ Q_i^{(2)}\mathcal{L}_{\xi q}^{(2)},\ Q_i^{(3)}\mathcal{L}_{\xi q}^{(1)}$	$\alpha_s(\mu_h)\,\alpha_s(\mu_i)/\pi$	$\alpha_s(\mu_i)$
$A^{(2)}$	$Q_i^{(5)}$	$\alpha_s(\mu_h)$	—

annihilation: “spectator” quark line ends at weak vertex

$Q^{(4)}$: local annihilation (ALRS)

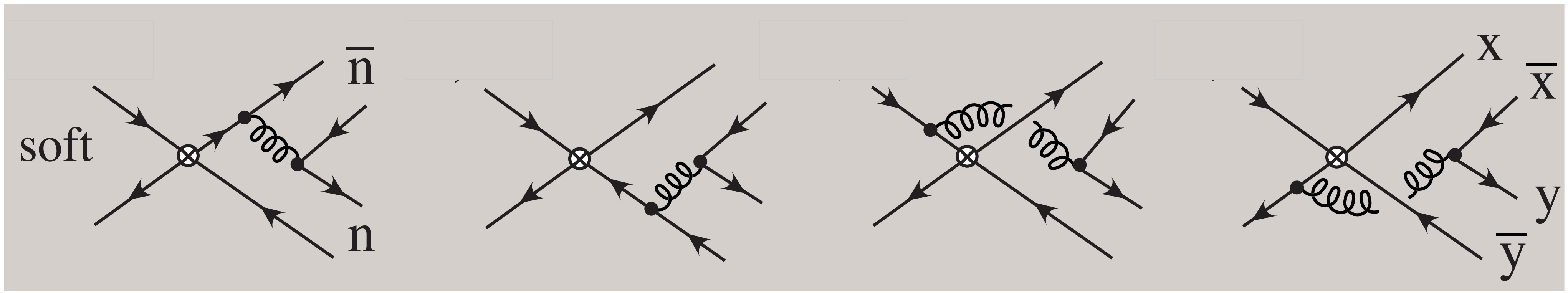
SCET$_{\text{I}}$:



SCET$_{\text{II}}$:

$$O_{1d}^{(1L)} = \frac{2}{m_b^3} \sum_q \left[\bar{d}_s P_R b_v\right] \left[\bar{u}_{\bar{n},\omega_2} \not{n} P_L \, q_{\bar{n},\omega_3}\right] \left[\bar{q}_{n,\omega_1} \not{\bar{n}} P_L u_{n,\omega_4}\right]$$

QCD:



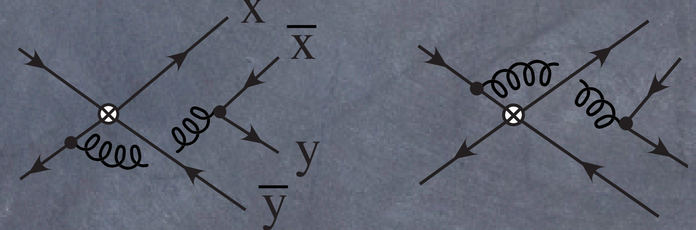
SCET$_{II}$:

$$
\begin{aligned}
O_{1d}^{(1L)} &= \frac{2}{m_b^3} \sum_q \left[\bar{d}_s P_R b_v\right]\left[\bar{u}_{\bar{n},\omega_2} \not{n} P_L\, q_{\bar{n},\omega_3}\right]\left[\bar{q}_{n,\omega_1} \not{\bar{n}} P_L u_{n,\omega_4}\right], \\
O_{2d}^{(1L)} &= \frac{2}{m_b^3} \sum_q \left[\bar{u}_s P_R b_v\right]\left[\bar{d}_{\bar{n},\omega_2} \not{n} P_L\, q_{\bar{n},\omega_3}\right]\left[\bar{q}_{n,\omega_1} \not{\bar{n}} P_L u_{n,\omega_4}\right], \\
O_{3d}^{(1L)} &= \frac{2}{m_b^3} \sum_{q,q'} \left[\bar{d}_s P_R b_v\right]\left[\bar{q}'_{\bar{n},\omega_2} \not{n} P_L\, q_{\bar{n},\omega_3}\right]\left[\bar{q}_{n,\omega_1} \not{\bar{n}} P_L q'_{n,\omega_4}\right], \\
O_{4d}^{(1L)} &= \frac{2}{m_b^3} \sum_{q,q'} \left[\bar{q}'_s P_R b_v\right]\left[\bar{d}_{\bar{n},\omega_2} \not{n} P_L\, q_{\bar{n},\omega_3}\right]\left[\bar{q}_{n,\omega_1} \not{\bar{n}} P_L q'_{n,\omega_4}\right], \\
O_{5d}^{(1L)} &= \frac{2}{m_b^3} \sum_q \left[\bar{d}_s P_R b_v\right]\left[\bar{u}_{\bar{n},\omega_2} \not{n} P_R\, q_{\bar{n},\omega_3}\right]\left[\bar{q}_{n,\omega_1} \not{\bar{n}} P_R\, u_{n,\omega_4}\right], \\
O_{6d}^{(1L)} &= \frac{2}{m_b^3} \sum_q \left[\bar{u}_s P_R b_v\right]\left[\bar{d}_{\bar{n},\omega_2} \not{n} P_R\, q_{\bar{n},\omega_3}\right]\left[\bar{q}_{n,\omega_1} \not{\bar{n}} P_R\, u_{n,\omega_4}\right], \\
O_{7d}^{(1L)} &= \frac{2}{m_b^3} \sum_{q,q'} \left[\bar{d}_s P_R b_v\right]\left[\bar{q}'_{\bar{n},\omega_2} \not{n} P_R\, q_{\bar{n},\omega_3}\right]\left[\bar{q}_{n,\omega_1} \not{\bar{n}} P_R\, q'_{n,\omega_4}\right], \\
O_{8d}^{(1L)} &= \frac{2}{m_b^3} \sum_{q,q'} \left[\bar{q}'_s P_R b_v\right]\left[\bar{d}_{\bar{n},\omega_2} \not{n} P_R\, q_{\bar{n},\omega_3}\right]\left[\bar{q}_{n,\omega_1} \not{\bar{n}} P_R\, q'_{n,\omega_4}\right]
\end{aligned}
$$

$$O_{1d}^{(1L)} = \frac{2}{m_b^3} \sum_q \left[\bar{d}_s P_R b_v\right]\left[\bar{u}_{\bar{n},\omega_2} \not{n} P_L\, q_{\bar{n},\omega_3}\right]\left[\bar{q}_{n,\omega_1} \not{\bar{n}} P_L u_{n,\omega_4}\right]$$

has coefficient function:

$$a_{1u} = \frac{C_F \pi \alpha_s(\mu_h)}{N_c^2}\left[\frac{1}{\bar{x}^2 y} - \frac{1}{y(x\bar{y}-1)}\right]_{\varnothing}\left(C_1 + \frac{3}{2}C_{10}\right)$$



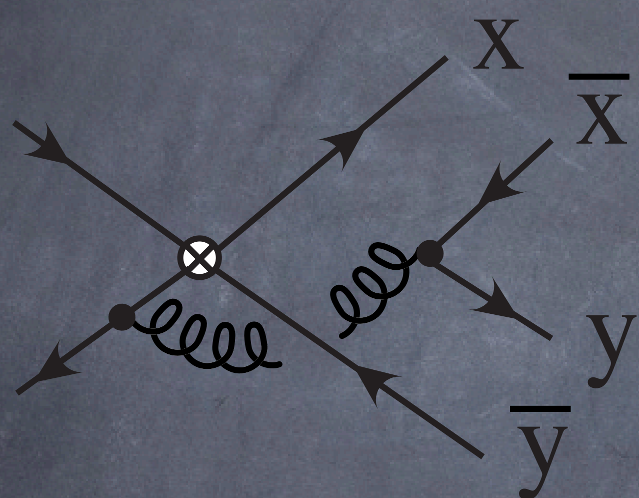
Divergent convolution?

finite and real by zero-bin subtraction in M(anohar)S(tewart)-rapidity factorization:

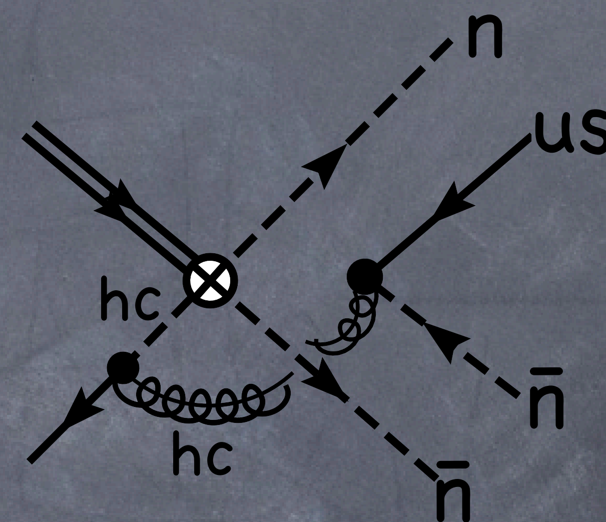
$$A \propto f_B \int dx\, dy \Big[\frac{1}{\bar{x}^2 y} - \frac{1}{y(x\bar{y}-1)}\Big]_{\varnothing} \phi_{M_1}(x)\phi_{M_2}(y) \in \Re$$

Where's the zero-bin contribution?

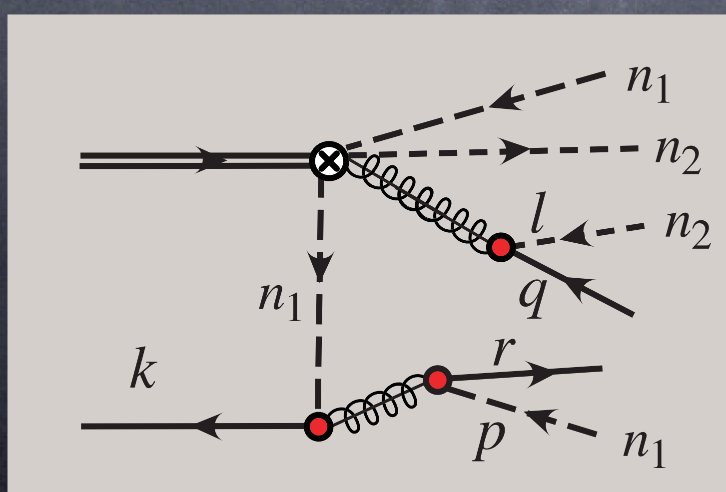
offending graph:



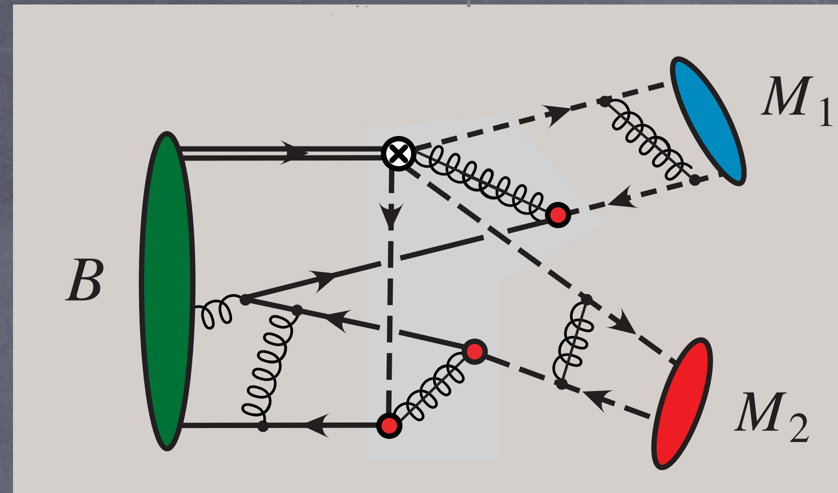
$\bar{x}$ in zero bin:



counted in T-products with $\left[\mathcal{L}^{(1)}_{\xi q}\right]^3$

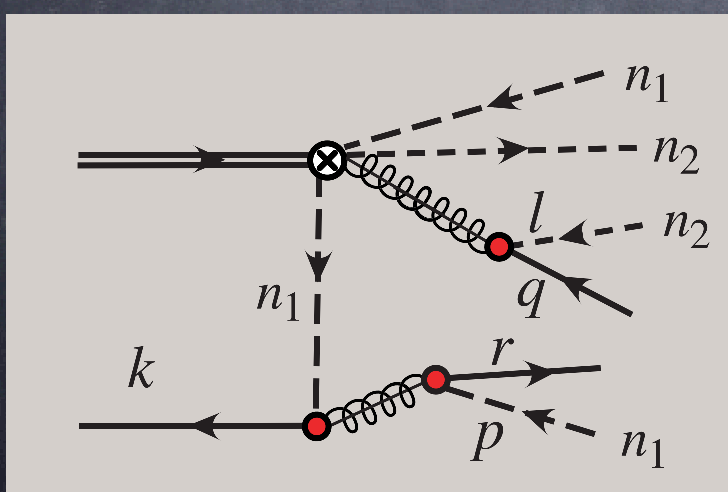


Complex annihilation



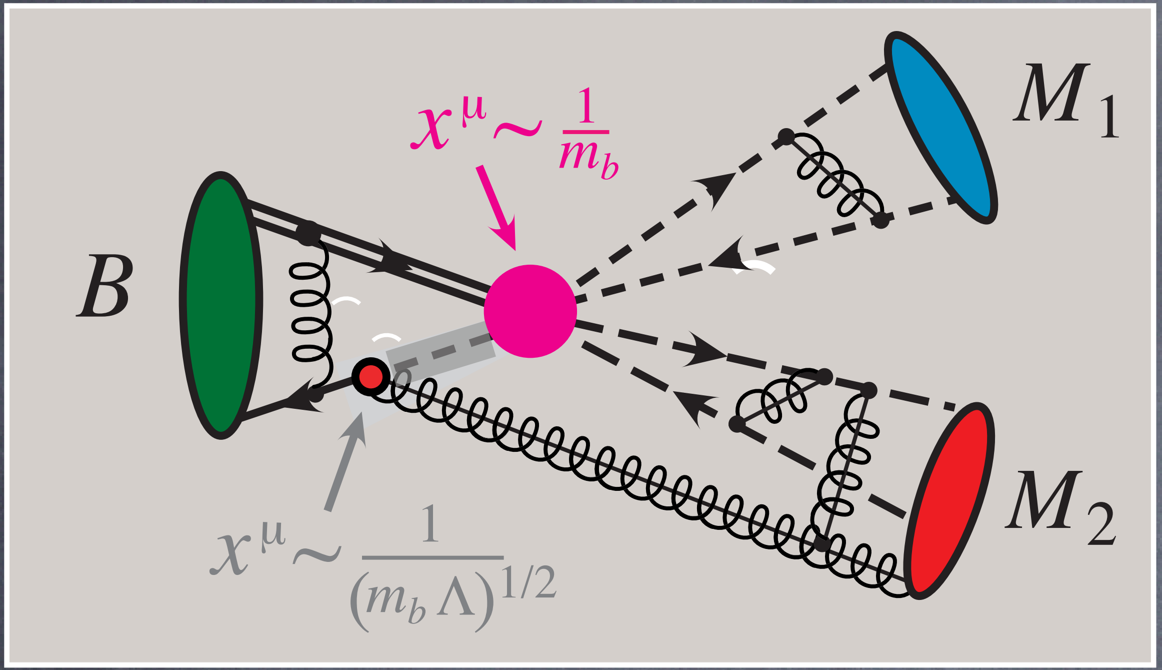
$\propto \alpha_s(\mu_i)^2$

$$A(\bar{B} \to M_1 M_2) = -i * \text{matching coefficients}$$
$$\otimes \langle M_1 | c_n \text{ fields} | 0 \rangle$$
$$\otimes \langle M_2 | c_{\bar{n}} \text{ fields} | 0 \rangle$$
$$\otimes \langle 0 | \text{soft fields} | \bar{B} \rangle$$



non-perturbative strong phase from incomplete cancellation of directed soft Wilson lines

$Q^{(2)}\mathcal{L}^{(1)}_{\xi q}$:three-parton annihilation (ARS)



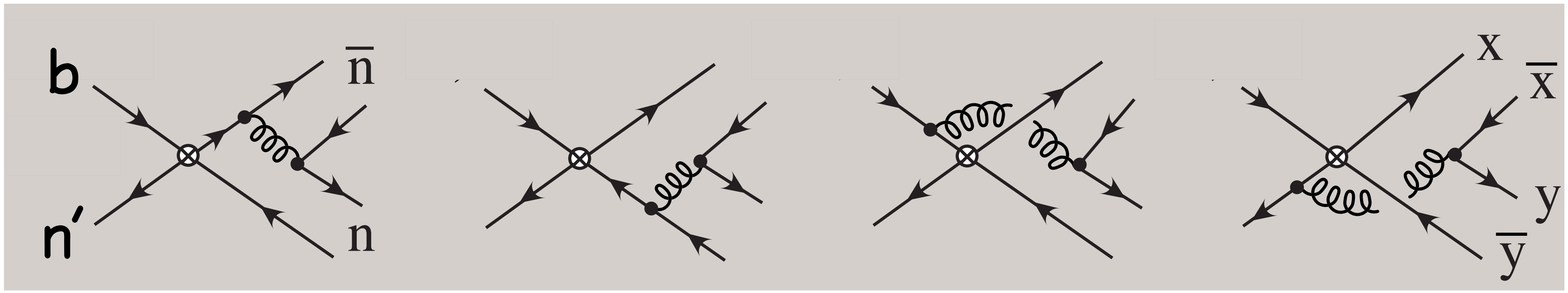
● = $\mathcal{L}^{(1)}_{\xi q} = \bar{q}'_{us} ig \not{\mathcal{B}}^{\perp}_{n'} q'_{n'}$

$$Q^{(2)}_{id} \propto [\bar{q}'_{n',\omega_5}\Theta_{us} b_v][\bar{d}_{\bar{n},\omega_2}\Theta_{\bar{n}} q_{\bar{n},\omega_3}][\bar{q}_{n,\omega_1}\Theta_n q'_{n,\omega_4}]$$

match ↓ onto $\mathrm{SCET_{II}}$

$$O^{(1T)}_{id} \propto \frac{1}{n'\cdot k}[\bar{q}'_{s,n'\cdot k}\Gamma_s b_v][\bar{d}_{\bar{n}}\Gamma_{\bar{n}} q_{\bar{n}}][\bar{q}_n \Gamma_n q'_n] ig\mathcal{B}^{\perp\beta}_{n'}$$

QCD:



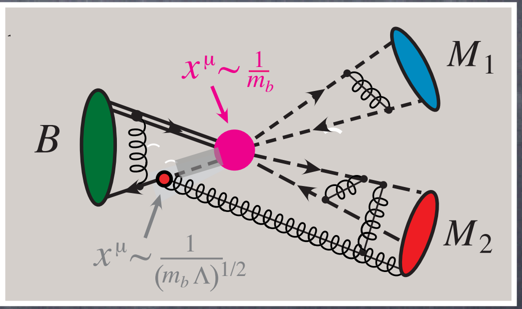
SCET$_{\mathrm{I}}$:

$$Q_{1d}^{(2)} = \frac{2}{m_b^3} \sum_{q,q'} \left[\bar{q}'_{n,\omega_5} P_L \gamma_\perp^\alpha T^a b_v\right] \left[\bar{d}_{\bar{n},\omega_2} \not{n} P_L \, q_{\bar{n},\omega_3}\right] \left[\bar{q}_{n,\omega_1} \not{\bar{n}} \, \gamma_\alpha^\perp T^a P_R \, q'_{n,\omega_4}\right],$$

$$Q_{2d}^{(2)} = \frac{2}{m_b^3} \sum_{q,q'} \left[\bar{q}'_{\bar{n},\omega_5} P_L \gamma_\perp^\alpha T^a b_v\right] \left[\bar{d}_{\bar{n},\omega_2} \not{n} \gamma_\alpha^\perp T^a \; P_R \, q_{\bar{n},\omega_3}\right] \left[\bar{q}_{n,\omega_1} \not{\bar{n}} P_R \, q'_{n,\omega_4}\right],$$

$$Q_{3d,4d}^{(2)} = Q_{1d,2d}^{(2)} \, \frac{3e_{q'}}{2}$$

+ similar with color $1 \otimes 1 \otimes 1$



SCET$_{\mathrm{II}}$:

$$O_{1d}^{(1T)} = \frac{1}{m_b^3 \, k^+} \sum_{q,q'} \left[\bar{q}'_{s,-k^+} P_L \not{n} \, S_n^\dagger b_v\right] \left[\bar{d}_{\bar{n},\omega_2} \not{n} P_L \, q_{\bar{n},\omega_3}\right] \left[\bar{q}_{n,\omega_1} \not{\bar{n}} \, (ig\not{\mathcal{B}}_\perp)_{n,\omega_5} P_R \, q'_{n,\omega_4}\right],$$

$$O_{2d}^{(1T)} = \frac{1}{m_b^3 \, k^-} \sum_{q,q'} \left[\bar{q}'_{s,-k^-} P_L \not{\bar{n}} \, S_{\bar{n}}^\dagger b_v\right] \left[\bar{d}_{\bar{n},\omega_2} \not{n} \, (ig\not{\mathcal{B}}_\perp)_{\bar{n},\omega_5} P_R \, q_{\bar{n},\omega_3}\right] \left[\bar{q}_{n,\omega_1} \not{\bar{n}} P_R \, q'_{n,\omega_4}\right],$$

$$O_{3-4d}^{(1T)} = O_{1-2d}^{(1T)} \, \frac{3e_{q'}}{2}$$

hard matching coefficients:

"polluted"

$$a_1^{hc}(x,y,\bar{y}) = \frac{\pi\alpha_s(m_b)}{N_C}\left\{\frac{2C_F C_5 + C_6}{y[x(1-y)-1]} + \frac{(2C_F - C_A)C_5 + C_6}{(1-x)y(1-\bar{y})}\right\},$$

$$a_2^{hc}(x,\bar{x},y) = \frac{\pi\alpha_s(m_b)}{N_C}\left\{-\frac{(2C_F - C_A)C_5 + C_6}{\bar{x}[(1-\bar{x})(1-y)-1]} - \frac{2C_F C_5 + C_6}{\bar{x}y(1-x)}\right\}$$

factorization theorem:

$$A^{(1ann)}_{\mathrm{hard-collin}} = \frac{G_F f_B m_B}{\sqrt{2}\, m_b N_c}\,(\lambda_u^{(d)} + \lambda_c^{(d)})\int_0^\infty dk\, \frac{\phi_B^+(k)}{k}$$
$$\times\left\{ f_{3M_1} f_{M_2}\int_0^1 dx\int_0^1 dy\int_0^{1-y} d\bar{y}\;\frac{H_{hc1}^{M_1M_2}(x,y,\bar{y})}{1-y-\bar{y}}\;\phi_{3M_1}(y,\bar{y})\phi_{M_2}(x)\right.$$
$$\left. + \eta_{M_1} f_{M_1} f_{3M_2}\int_0^1 dy\int_0^1 dx\int_0^{1-x} d\bar{x}\;\frac{H_{hc2}^{M_1M_2}(x,\bar{x},y)}{1-x-\bar{x}}\;\phi_{M_1}(y)\phi_{3M_2}(x,\bar{x})\right\}$$

Numerical estimates

- simple model for distribution amplitudes
- non-perturbative parameters from QCD sum rules and lattice QCD
- large errors

$$\frac{|A^{(1)}_{Lann}(K^-\pi^+)|}{|A^{(1)}_{Expt.Penguin}(K\pi)|} \approx 0.1 \pm 0.1$$

three parton $\approx$ local

Conclusion

eliminated a possible source of large penguins:
annihilation is factorizable and real