SCET Workshop

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A complete subleading shape functions analysis for $\bar{B} \to X_s \gamma$

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Based on hep-ph/0609224, and work on progress

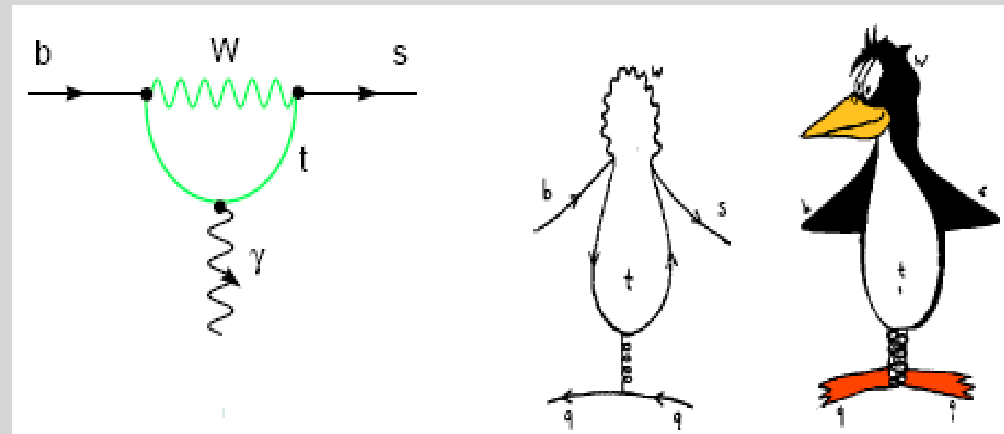
Outline

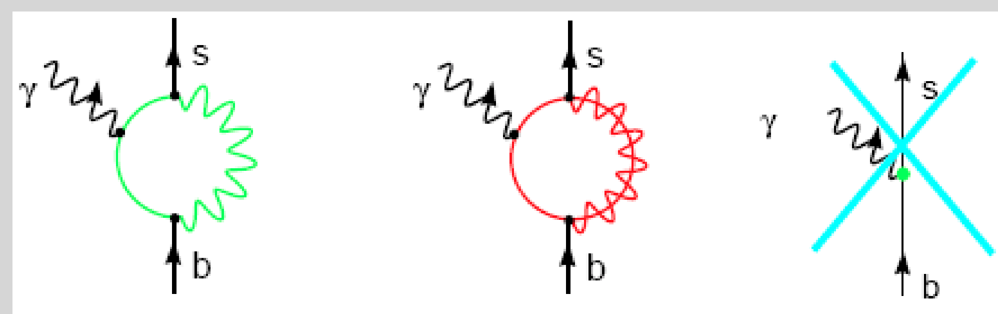
- Introduction
- Classification of Subleading Shape Functions
- NLO matching of H_{eff} to SCET
- Q7-Q8 non-local enhancement
- Q8-Q8
- Q7-Q1 (4-quark operators)
- Summary

Introduction

- Precision studies of inclusive $\bar{B}$-meson decays : cornerstone of quark flavor physics in and beyond SM
- Rare decays $\bar{B} \to X_s \gamma$
  - important for $|V_{ub}|$, etc.
  - complementary to collider physics

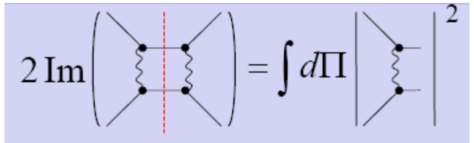
    (bound on top quark mass and Higgs mass)
  - sensitive to higher scales (constrains on model building)





Introduction

- Inclusive decay: optical theorem



$$\Gamma = \frac{1}{2M_B}\sum_f \int \frac{d^3P_f}{2E_f(2\pi)^3}|\mathcal{M}(i\to f)|^2(2\pi)^4\delta^4(P_i-P_f)$$

$$= \frac{1}{M_B}\mathrm{Im}\,\mathcal{M}(i\to i), \qquad \mathcal{M} = i\sum_{j,k}\int d^4x\,\underbrace{\langle \bar{B}|\,T\{Q_j^\dagger(0),\,Q_k(x)\}|\bar{B}\rangle}$$

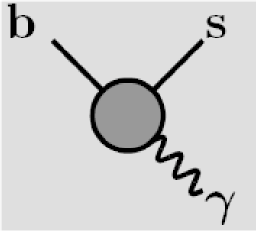
Forward B-meson matrix elements of local operators

- Effective Weak Hamiltonian

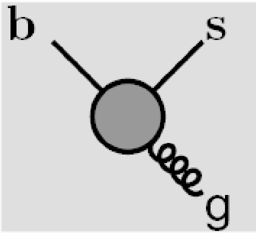
$$\Gamma \sim \mathrm{Im}\,\langle \bar{B}|\mathcal{H}_{eff}\,\mathcal{H}_{eff}|\bar{B}\rangle$$

$$\langle X_s\gamma|\mathcal{H}_{eff}|\,\bar{B}\rangle = \sum_{i=1\ldots8} C_i(\mu)\langle\, X_s\gamma|Q_i(\mu)\,|\bar{B}\rangle$$

$\mathcal{O}_{7\gamma}$



$\mathcal{O}_{8g}$



- Our notation: "Q_j-Q_k contribution"
- Describe b→s transitions by an effective Hamiltonian
- New physics shows up as modified C_i,

  (or as new operators)

OPE Region

- Total decay rate has been calculated in an expansion about heavy quark limit using local OPE. (leading power correction $\sim(\Lambda/M_B)^2$)
- Perturbative QCD corrections require resummation of large logarithms $\rightarrow$ a lot of efforts from global community

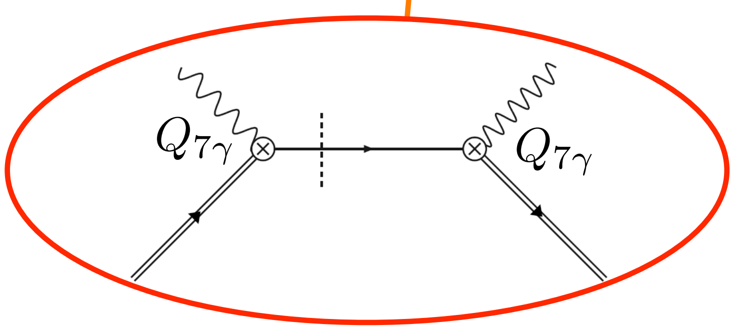
New: non-local effect

$$\Gamma\left(B \rightarrow X_s \gamma\right) = \frac{G_F^2 \alpha}{32\pi^4} \left|V_{tb} V_{ts}^*\right|^2 m_b^5 \left|C_{7\gamma}\right|^2$$

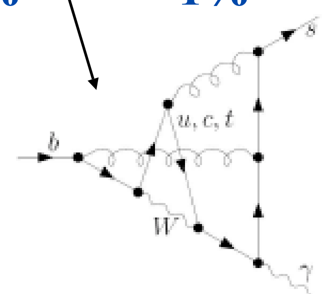
$$\times \left\{ 1 + \mathcal{O}(\alpha_s) + \mathcal{O}(\alpha_s^2) + \mathcal{O}(\alpha_{\rm em}) + \overbrace{\mathcal{O}\left(\frac{\Lambda^2}{m_b^2}\right) + \mathcal{O}\left(\frac{\Lambda^2}{m_c^2}\right)}^{\text{Calculable using local OPE}} + \mathcal{O}\left(\frac{\alpha_s \Lambda}{m_b}\right) \right\}$$

NLO **NNLO**

~25% **~7%** **~1%** **~3%** **~1%**



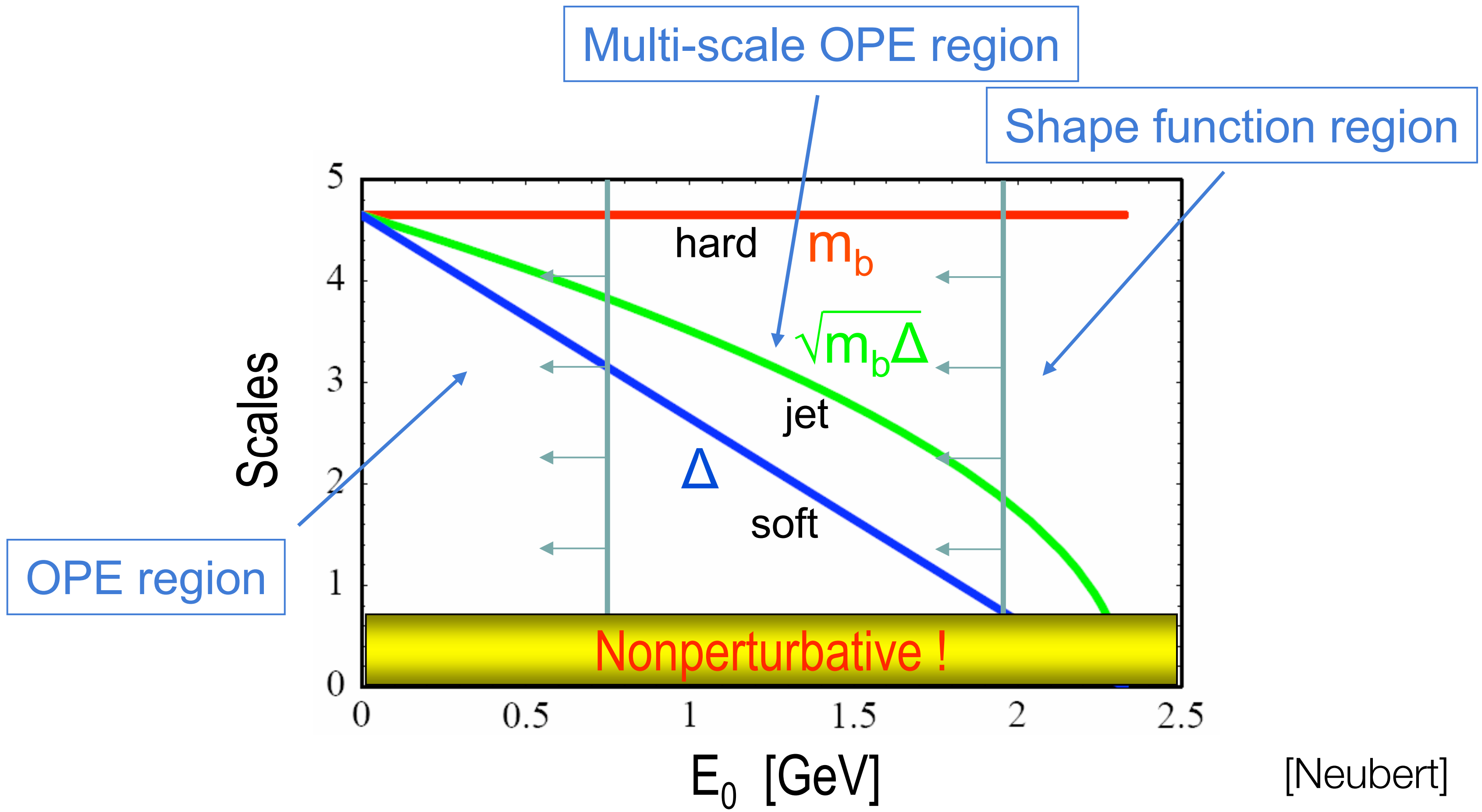
LO from T{$Q_{7\gamma}$,$Q_{7\gamma}$}



+ O(1000) similar graphs + 4-loop anomalous dimensions, etc…

Local OPE doesn't have this order of power corrections

Event fraction at NNLO



[Neubert]

Shape-function Region

- OPE breaks down for differential inclusive decay distribution near phase space boundary (E_γ is near kinematical endpoint): most accessible to experiment

- Shape function method is needed to account for non-perturbative effect (a twist expansion involving forward matrix elements of non-local light-cone operators)

- At lowest order in the $1/m_b$ expansion, it is related to the leading order shape function which is a matrix element of a non local operator $\langle B|\bar{h}_v(0)[0,x]h_v(x)|B\rangle$

- SCET (Soft Collinear Effective Theory) is a relevant tool

Subleading Shape Functions

- In the SCET based calculation, the (subleading) shape functions are calculated via a two step matching. QCD → SCET → HQET

  - In the first step QCD is matched onto SCET and the hard function is extracted.

  - In the second step SCET is matched onto HQET and the jet function is extracted.

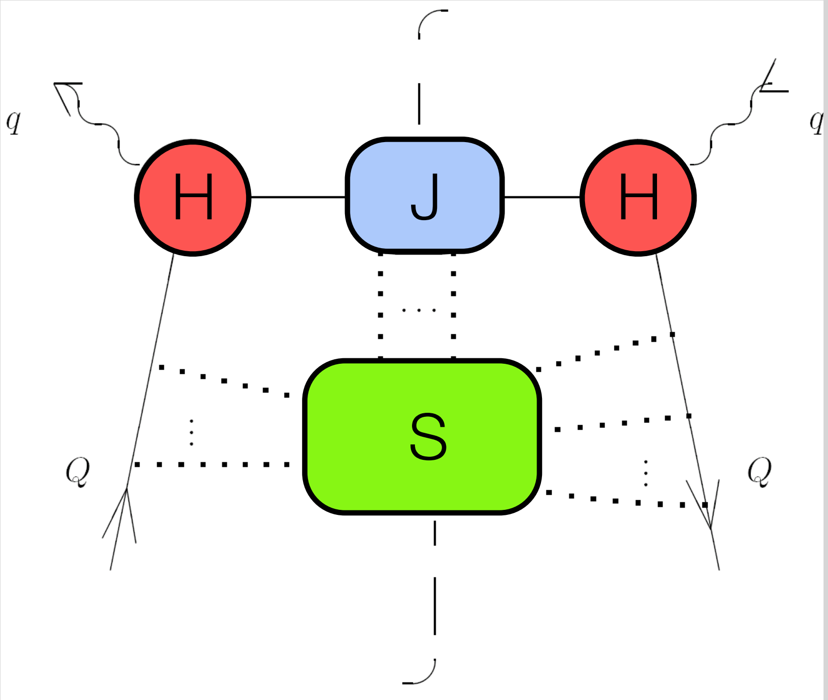
$$U_y(\mu_h, \mu_i)\, H(y, \mu_h) \int_0^{P_+} d\hat{\omega}\; m_b J(y, m_b(P_+ - \hat{\omega}), \mu_i)\, \hat{S}(\hat{\omega}, \mu_i)$$

    - Beyond leading order in $1/m_b$, the hard, jet, and shape functions are not unique.

    - Subleading hard functions are only known at tree level and it is sufficient to analyze the subleading shape functions at tree level.

Factorization theorem

- At Λ_{QCD}/m_b, the decay rate factorizes into a convolution of three objects:
  - H - physics at scale $\mu \geq m_b$
    Calculable in PT
  - J - physics at scale $\mu \sim \sqrt{m_b \Lambda}$
    Calculable in PT
  - S - physics at scale $\mu \sim \Lambda_{QCD}$
    Non-perturbative function



[Neubert
Korchemsky, Sterman
Bauer, Pirjol, Stewart]

$$\frac{d\Gamma}{dE} \sim H_s \cdot J \otimes S + \cdots$$

[Korchemsky, Sterman
Bauer, Pirjol, Stewart]

Factorization beyond LO (subleading shape function)

- Current status of subleading shape function

$\bar{B} \to X_u l^- \bar{\nu}$ and the $Q_{7\gamma} - Q_{7\gamma}$ subleading shape functions were classified

- Factorization beyond LO:

$$\frac{d\Gamma}{dE} \sim H_s \cdot J \otimes S + \frac{1}{m_b} \sum_k h_s^k \cdot j_s^k \otimes s_s^k + \cdots$$

[Lee, Stewart
Bosch, Neubert, Paz
Beneke, Campanario, Mannel, Pecjak]

- What about other operators for $\bar{B} \to X_s \gamma$?

i.e. Q_j-Q_k

remaining part of this talk

Classification of Subleading Shape Functions

- In order to systematically analyze the subleading shape functions, one has to match the entire weak Hamiltonian onto SCET.
- The Hamiltonian needs to be matched up to second order in the SCET expansion in order to extract the subleading shape functions at order $1/m_b$.
- From the entire weak Hamiltonian only a few operators will give subleading shape functions which are not loop suppressed.
- The subleading shape functions that are not loop suppressed arise from the following contributions:

$$Q_{7\gamma} - Q_{8g},\ Q_{8g} - Q_{8g},\ \text{and}\ Q_1 - Q_{7\gamma}$$

NLO matching of $\mathcal{H}_{\text{eff}}$ to SCET

- The effective weak Hamiltonian

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \left(\lambda_p C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3,\ldots,6} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \right) + \text{h.c.},$$

where $\lambda_p = V_{ps}^* V_{pb}$, and

$$\begin{aligned}
Q_1^p &= (\bar{p}b)_{V-A}(\bar{s}p)_{V-A}\,, & Q_2^p &= (\bar{p}_i b_j)_{V-A}(\bar{s}_j p_i)_{V-A}\,, \\
Q_3 &= (\bar{s}b)_{V-A} \textstyle\sum_q (\bar{q}q)_{V-A}\,, & Q_4 &= (\bar{s}_i b_j)_{V-A} \textstyle\sum_q (\bar{q}_j q_i)_{V-A}\,, \\
Q_5 &= (\bar{s}b)_{V-A} \textstyle\sum_q (\bar{q}q)_{V+A}\,, & Q_6 &= (\bar{s}_i b_j)_{V-A} \textstyle\sum_q (\bar{q}_j q_i)_{V+A}\,,
\end{aligned}$$

$$Q_{7\gamma} = \frac{-e}{8\pi^2} m_b \bar{s}\sigma_{\mu\nu}(1+\gamma_5) F^{\mu\nu} b, \quad Q_{8g} = \frac{-g_s}{8\pi^2} m_b \bar{s}\sigma_{\mu\nu}(1+\gamma_5) G^{\mu\nu} b.$$

NLO matching of $\mathcal{H}_{\rm eff}$ to SCET

- **Possible operator basis**: We refer the photon as a **"anti-hard-collinear" (ahc)** and the hadronic jet **"hard-collinear" (hc)**.

- Start by listing the fields and their scales in terms of $\lambda = \Lambda_{\rm QCD}/m_b$

hc fields: quark $\xi \sim \lambda^{1/2}$
gluon $A^\mu_{\rm hc} \sim (\lambda, 1, \lambda^{1/2})$
derivative $i\partial^\mu_{\rm hc} \sim (\lambda, 1, \lambda^{1/2})$

ahc fields: quark $\chi \sim \lambda^{1/2}$
gluon $A^\mu_{\overline{\rm hc}} \sim (1, \lambda, \lambda^{1/2})$
derivative $i\partial^\mu_{\overline{\rm hc}} \sim (1, \lambda, \lambda^{1/2})$
photon $A^{\rm em}_\perp \sim \lambda^{1/2}$

- notation:

$a^\mu \sim (n \cdot a, \bar{n} \cdot a, a_\perp)$

soft fields: quark $q_s \sim \lambda^{3/2}$
heavy quark $h \sim \lambda^{3/2}$
gluon $A^\mu_s \sim (\lambda, \lambda, \lambda)$
derivative $i\partial^\mu_s \sim (\lambda, \lambda, \lambda)$

Possible operator basis

- In constructing the possible operators we assume that the final state can only contain one ahc photon

- All the other particles need to be hc or soft, but we can have several ahc and/or soft particles that will convert to an ahc photon

Operators containing photon: $\lambda = \Lambda_{\mathrm{QCD}}/m_b$

$\mathcal{O}(\lambda^{5/2})$:	$\bar{\xi} A_\perp^{\mathrm{em}} h$
$\mathcal{O}(\lambda^{3})$:	$\bar{\xi} A_\perp^{\mathrm{hc}} A_\perp^{\mathrm{em}} h, \quad \bar{\xi} \partial_\perp^{\mathrm{hc}} A_\perp^{\mathrm{em}} h$
$\mathcal{O}(\lambda^{7/2})$:	$\bar{\xi} A_\perp^{\mathrm{hc}} A_\perp^{\mathrm{hc}} A_\perp^{\mathrm{em}} h, \quad \bar{\xi} (A_\perp^{\mathrm{hc}} \partial_\perp^{\mathrm{hc}}) A_\perp^{\mathrm{em}} h, \quad \bar{\xi} \partial_\perp^{\mathrm{hc}} A_\perp^{\mathrm{hc}} A_\perp^{\mathrm{em}} h, \quad \bar{\xi} \partial_\perp^{\mathrm{hc}} \partial_\perp^{\mathrm{hc}} A_\perp^{\mathrm{em}} h$ $\bar{\xi} n \cdot A^{\mathrm{hc}} A_\perp^{\mathrm{em}} h, \quad \bar{\xi} n \cdot \partial^{\mathrm{hc}} A_\perp^{\mathrm{em}} h, \quad \bar{\xi} A^{\mathrm{s}} so A_\perp^{\mathrm{em}} h, \quad \bar{\xi} A_\perp^{\mathrm{em}} \partial^{\mathrm{s}} h$ $\bar{\xi} A_\perp^{\mathrm{em}} h \bar{\xi} \xi$

Operators without photon

-need Lagrangian insertion: cost power counting in λ

1. (a) $\mathcal{O}(\lambda^{1/2}):\quad \chi(\bar\chi) \to A_\perp^{\mathrm{em}} + q_s$

   (b) $\mathcal{O}(\lambda^{1}):\quad A_\perp^{\overline{\mathrm{hc}}} \to A_\perp^{\mathrm{em}} + q_s + \bar q_s,\ A_\perp^{\overline{\mathrm{hc}}} \to A_\perp^{\mathrm{em}} + A_s + A_s$

2. (a) $\mathcal{O}(\lambda^{0}):\quad \bar\chi\chi \to A_\perp^{\mathrm{em}}$

   (b) $\mathcal{O}(\lambda^{1/2}):\quad \bar\chi\chi \to A_\perp^{\mathrm{em}} + A_s$

   (c) $\mathcal{O}(\lambda^{1/2}):\quad \chi(\bar\chi)A_\perp^{\overline{\mathrm{hc}}} \to A_\perp^{\mathrm{em}} + q_s$

   (d) $\mathcal{O}(\lambda^{1/2}):\quad A_\perp^{\overline{\mathrm{hc}}}A_\perp^{\overline{\mathrm{hc}}} \to A_\perp^{\mathrm{em}} + A_s$

3. (a) $\mathcal{O}(\lambda^{0}):\quad \bar\chi\chi A_\perp^{\overline{\mathrm{hc}}} \to A_\perp^{\mathrm{em}}$

   (b) $\mathcal{O}(\lambda^{1/2}):\quad A_\perp^{\overline{\mathrm{hc}}}A_\perp^{\overline{\mathrm{hc}}}A_\perp^{\overline{\mathrm{hc}}} \to A_\perp^{\mathrm{em}},$

$\mathcal{O}(\lambda^3)$: $\quad \bar\xi h \bar\chi\chi.$

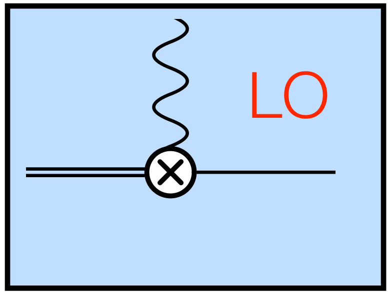
$\mathcal{O}(\lambda^{7/2})$: $\quad \bar\xi A_\perp^{\overline{\mathrm{hc}}} h$

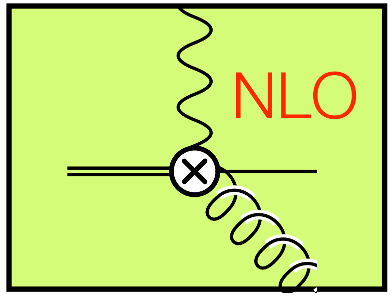
$\bar\xi h \bar\xi\chi, \quad \bar\xi h\bar\chi\xi, \quad \bar\xi A_\perp^{\overline{\mathrm{hc}}}A_\perp^{\overline{\mathrm{hc}}} h$

$\bar\xi (A_\perp^{\overline{\mathrm{hc}}})^3 h,\ \bar\xi A_\perp^{\mathrm{hc}} h\bar\chi\chi,\ \bar\xi h\bar\chi A_\perp^{\overline{\mathrm{hc}}}\chi,\ \bar\xi\partial_\perp^{\mathrm{hc}} h\bar\chi\chi,\ \bar\xi h\bar\chi\partial_\perp^{\overline{\mathrm{hc}}}\chi,\ \bar\xi h\bar\chi\partial_\perp^{\overline{\mathrm{hc}}}\chi$

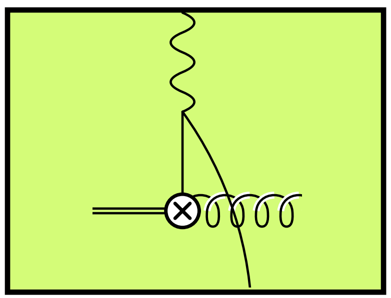
Subleading shape functions that are not loop suppressed

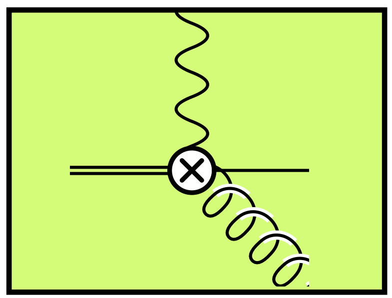
$Q_{7\gamma}$:

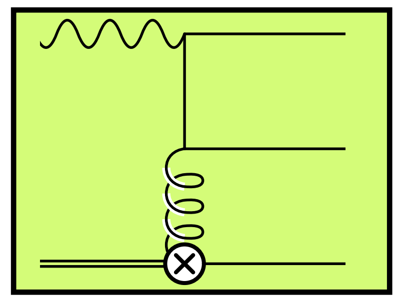




Q_{8g}:

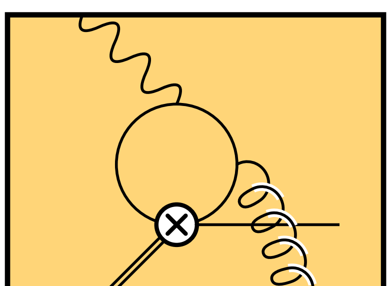






NLO

Q_{1c}:



NNLO
and C_1 is large

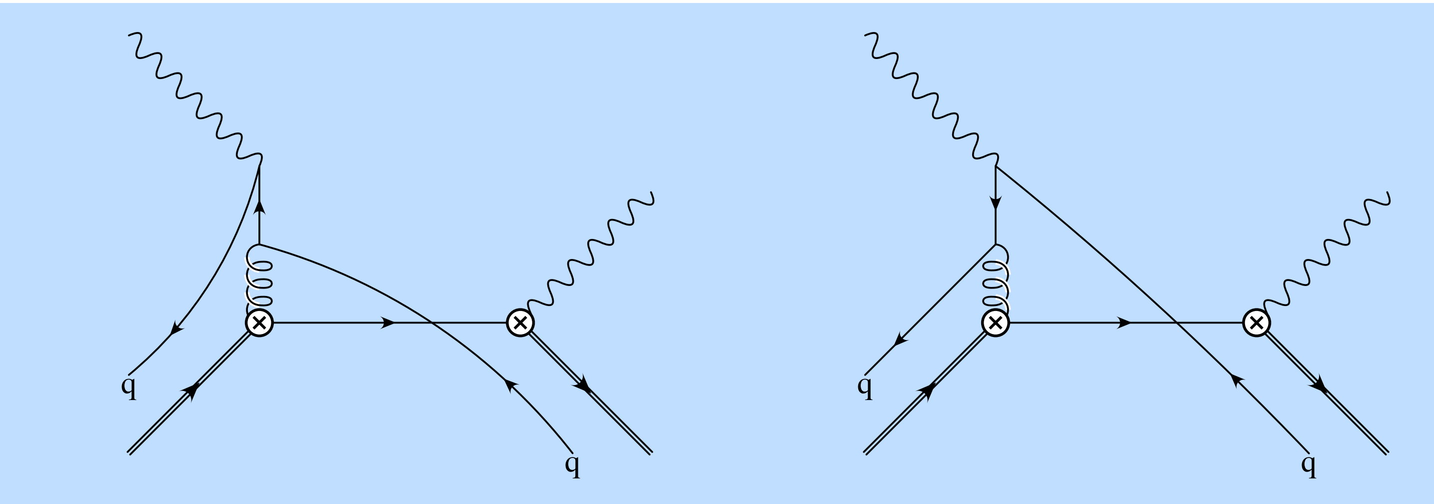
$\mathcal{O}(\lambda^{5/2})$: LO

$\mathcal{O}(\lambda^{3})$: NLO

$\mathcal{O}(\lambda^{7/2})$: NNLO

$Q_{7\gamma}$-Q_{8g}

Non-local operators from ahc gluon



- give rise to 4-quark operators containing not only s quarks, but also light u,d quarks

Subleading shape function

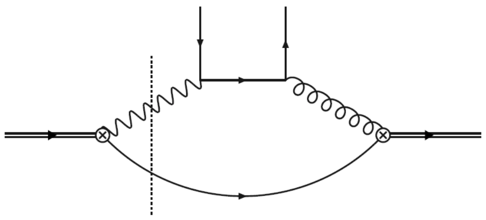
$$\frac{d\Gamma_{78a}}{dE_\gamma} = \frac{G_F^2 \alpha |V_{tb} V_{ts}^*|^2 C_{7\gamma}^2 m_b^2}{2\pi^4} \left(E_\gamma^2 \pi \alpha_s \frac{C_{8g}}{C_{7\gamma}} \int d\omega \, \delta(n \cdot p + \omega) f_{78a}(\omega) \right)$$

$$\int d\omega \, e^{-\frac{i}{2}\omega \bar{n} \cdot x} f_{78a}(\omega) = \frac{-1}{M_B} \sum_q e_q \left\{ \int_{-\infty}^0 dy_+ \int_{-\infty}^0 dz_+ \langle \bar{B} | \bar{h}(0) \cdots \bar{q}(z_+) q(y_+) \cdots h(x_-) | \bar{B} \rangle \right\}$$

$Q_{7\gamma}$-Q_{8g} (non-local enhancement)

- Contribution to total rate: (parameterized by matrix element of tri-local operators)

$$\Delta\Gamma = -\Gamma_{77}\frac{C_{8g}}{C_{7\gamma}}\frac{4\pi\alpha_s}{N_c m_b}\int_{-\infty}^{0} ds \int_{-\infty}^{0} dt \langle \bar{B}|\, C_F\,(O_1+O_2) - (T_1+T_2)\,|\bar{B}\rangle$$



Non-local operators

$$\left\{\begin{aligned} O_1 &= \sum_q e_q\, \bar{h}_v(0)\,\Gamma_R\, q(t\bar{n})\;\bar{q}(s\bar{n})\,\Gamma_R\, h_v(0)\,, \\ O_2 &= \sum_q \frac{e_q}{2}\, \bar{h}_v(0)\,\Gamma_R\gamma_{\perp\alpha}\, q(t\bar{n})\;\bar{q}(s\bar{n})\,\gamma_\perp^\alpha\Gamma_R\, h_v(0)\,, \\ T_1 &= \sum_q e_q\, \bar{h}_v(0)\,\Gamma_R\, t_a q(t\bar{n})\;\bar{q}(s\bar{n})\,\Gamma_R\, t_a h_v(0)\,, \\ T_2 &= \sum_q \frac{e_q}{2}\, \bar{h}_v(0)\,\Gamma_R\gamma_{\perp\alpha}\, t_a q(t\bar{n})\;\bar{q}(s\bar{n})\,\gamma_\perp^\alpha\Gamma_R\, t_a h_v(0)\,, \end{aligned}\right.$$

$$\Gamma_R = \not{\bar{n}}\,(1+\gamma_5)/2$$

§Due to the interference with $Q_{7\gamma}$,the effect is centered at large photon energy (cannot be eliminated by a cut)

§Note that in $\bar{B} \to X_s\gamma$, one does not sum over all the cut

§Process is not really inclusive (partonic sub-structure of photon)

$Q_{7\gamma}$-Q_{8g} (non-local enhancement)

- Model estimates:
  - Reliable field-theoretical estimates of these effects are very difficult to obtain.
  - In particular, lattice QCD is unable to handle operators with component fields separated by light-like distances
- Naive dimensional analysis suggests that

$\Delta\Gamma/\Gamma_{77} \sim (C_{8g}/C_{7\gamma})\,\pi\alpha_s(\Lambda/m_b)$, which could easily amount to a 5% correction to the decay rate.

- In more traditional applications of the OPE to inclusive B-meson decays, four-quark operators contribute at order $(\Lambda/m_b)^3$ in the heavy-quark expansion.
- The non-local operators lead to enhanced power corrections of order Λ/m_b, because the two ``vertical'' propagators have virtualities of order m_b/Λ and so introduce two powers of soft

$Q_{7\gamma}$-Q_{8g} (non-local enhancement)

- Vacuum insertion approximation $|0\rangle\langle 0|$
  - motivated by large-N_c counting rules.
  - well tested for local 4-quark operator in the analysis of *B*-hadron lifetimes.
  - Matrix elements of the operators $\mathbf{O_2}$ and $\mathbf{T_{\{1,2\}}}$ vanish in the VIA, either due to the color-octet structure of the quark bilinear $\mathbf{T_{\{1,2\}}}$ or due to the fact that there is no external perpendicular Lorentz vector available ($\mathbf{O_2}$ and $\mathbf{T_{\{1,2\}}}$): $\mathbf{O_1}$ **is the only operator contribute in VIA approximation**

$$\langle \bar{B}| \, O_1 \, |\bar{B}\rangle_{\text{VIA}} = e_q \, \frac{f_B^2 m_B}{4} \, \tilde{\phi}_+^B(s) \, [\tilde{\phi}_+^B(t)]^*$$

  - The integral over the position-space distribution amplitude can be evaluated to yield

$$-i \int_{-\infty}^{0} ds \, \tilde{\phi}_+^B(s) = \int_0^\infty \frac{d\omega}{\omega} \, \phi_+^B(\omega) = \frac{1}{\lambda_B} \, ,$$

$Q_{7\gamma}$-Q_{8g} (non-local enhancement)

- Vacuum Insertion Approximation

$$\frac{\Delta\Gamma_{\mathrm{VIA}}}{\Gamma_{77}} \approx -0.26 e_q \left(\frac{f_B}{\lambda_B}\right)^2 \approx -0.05 e_q \left(\frac{\lambda_B}{0.5\ \mathrm{GeV}}\right)^{-2}$$

  - where e_q=2/3 for decays of B^- mesons, while e_q =-1/3 for decays of mesons. Effect between –0.3 e_q % and –19 e_q % (depending on the value of inverse moment of B-meson LCDA)

- Flavor-dependent rate asymmetry

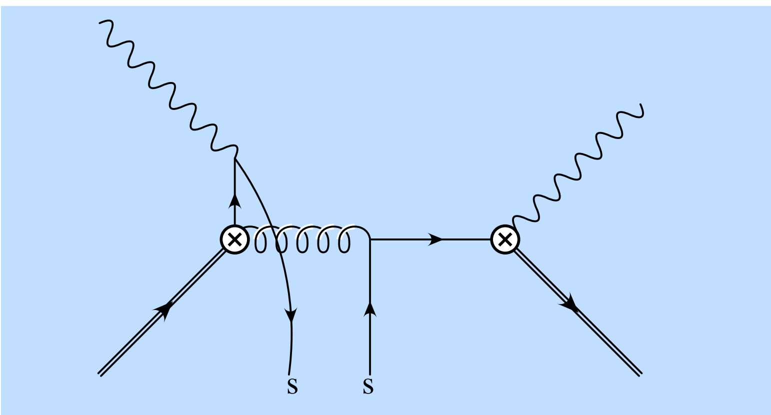
$$\frac{\Gamma(B^- \to X_s\gamma) - \Gamma(\bar{B}^0 \to X_s\gamma)}{\Gamma(\bar{B} \to X_s\gamma)} \approx -0.05 \left(\frac{\lambda_B}{0.5\ \mathrm{GeV}}\right)^{-2}$$

  - this amount to an effect between -2% and -19%, which is consistent with the recent BarBar measurement:

$$(1.2 \pm 11.6 \pm 1.8 \pm 4.8)\%$$

$Q_{7\gamma}$-Q_{8g}

Operators from hc gluon



$$\int d\omega\, e^{-\frac{i}{2}\omega\bar{n}\cdot x} f_{78b}(\omega) = \frac{1}{2M_B}\Big\{\Big[\int_{x_-}^{0} dy_- \int_{-\infty}^{0} dz_+$$

$$\Big(\langle \bar{B}|\bar{h}(0)\not{n}\not{\bar{n}} N^A(0,0) S_{\bar{n}}(0,0)\mathcal{Q}_{\bar{n}}(0,z_+)\bar{\mathcal{Q}}_n(y_-,0)\not{n}\not{\bar{n}} T^A S_n^\dagger(x_-,0)h(x_-)|\bar{B}\rangle$$

$$-\ \langle \bar{B}|\bar{h}(0)(\gamma_5)\not{n}\not{\bar{n}} N^A(0,0) S_{\bar{n}}(0,0)\mathcal{Q}_{\bar{n}}(0,z_+)\bar{\mathcal{Q}}_n(y_-,0)\not{n}\not{\bar{n}}(\gamma_5) T^A S_n^\dagger(x_-,0)h(x_-)|\bar{B}\rangle\Big)\Big]$$

$$+\ \Big[\int_{-x_-}^{0} dy_- \int_{-\infty}^{0} dz_+$$

$$\langle \bar{B}|\bar{h}(0)\not{n}\not{\bar{n}} N^A(0,0) S_{\bar{n}}(0,0)\mathcal{Q}_{\bar{n}}(0,z_+)\bar{\mathcal{Q}}_n(y_-,0)\not{n}\not{\bar{n}} T^A S_n^\dagger(-x_-,0)h(-x_-)|\bar{B}\rangle^*$$

$$-\ \langle \bar{B}|\bar{h}(0)(\gamma_5)\not{n}\not{\bar{n}} N^A(0,0) S_{\bar{n}}(0,0)\mathcal{Q}_{\bar{n}}(0,z_+)\bar{\mathcal{Q}}_n(y_-,0)\not{n}\not{\bar{n}}(\gamma_5) T^A S_n^\dagger(-x_-,0)h(-x_-)|\bar{B}\rangle^*\Big)\Big]$$

Subleading shape function

$$\frac{d\Gamma_{78b}}{dE_\gamma} = \frac{G_F^2\alpha|V_{tb}V_{ts}^*|^2 C_{7\gamma}^2 m_b^2}{2\pi^4}\left(\frac{E_\gamma^2}{4} e_s \pi\alpha_s \frac{C_{8g}}{C_{7\gamma}}\int d\omega\,\delta(n\cdot p+\omega) f_{78b}(\omega)\right)$$

- usual $1/m_b^2$ corrections
- contribution vanish in the VIA: soft quarks in the shape function are strange quarks

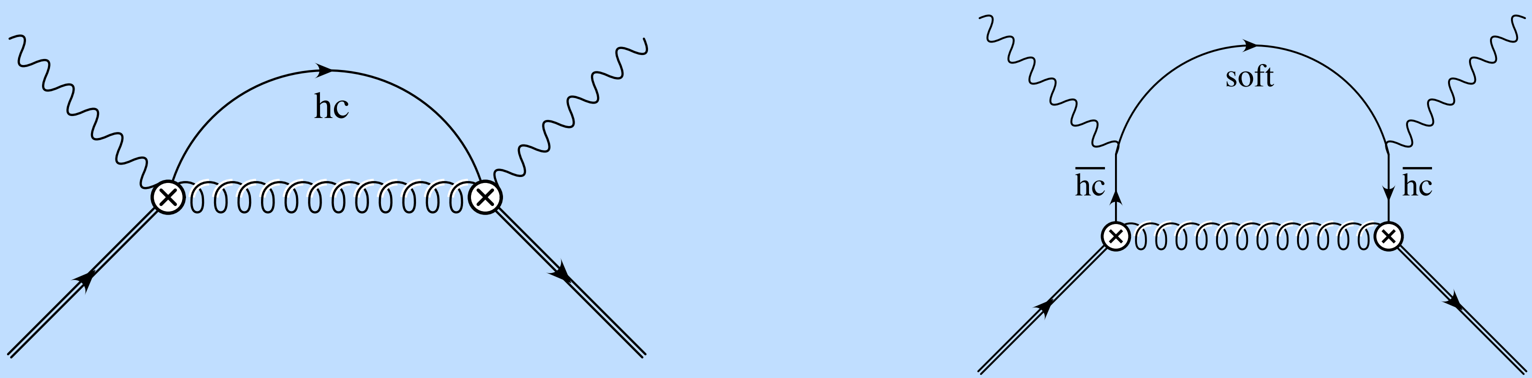
Q_{8g}-Q_{8g}

Relation to previous calculations

$$\left.\frac{d\Gamma_{88}}{dE_\gamma}\right|_{E_\gamma\to m_b/2} = \Gamma_{77}\cdot\frac{C_8^2}{9C_7^2}\frac{\alpha_s C_F}{4\pi}\frac{1}{m_b}\left\{-6-4\ln\left[\frac{m_s^2}{m_b(m_b-2E_\gamma)}\right]\right\}$$

[Ali, Gerub
Kapustin, Ligeti, Politzer]

We should be able to recover this result by using SCET



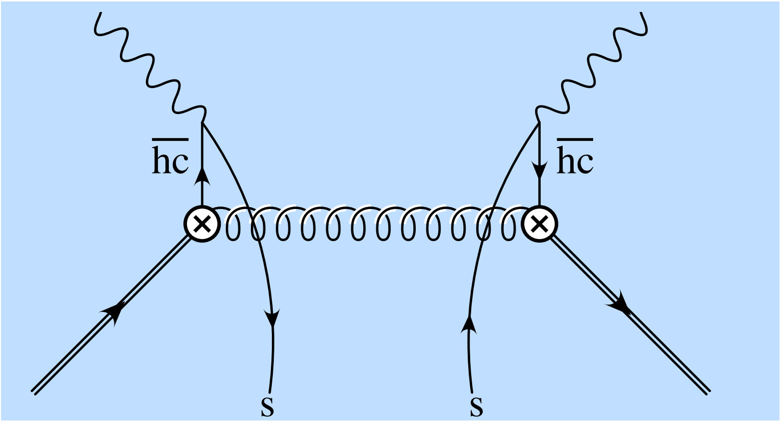
$$\left.\frac{d\Gamma_{88}^{hc}}{dE_\gamma}\right|_{E_\gamma\to m_b/2} = \Gamma_{77}\cdot\frac{C_8^2}{9C_7^2}\frac{\alpha_s C_F}{4\pi}\frac{2}{m_b}\frac{\langle\bar B|\bar h(1-\gamma_5)\not{n}h|\bar B\rangle}{M_B}\times\left\{-\frac{1}{\epsilon}+\frac{1}{2}+\gamma_E-\ln 4\pi-\ln\left[\frac{\mu^2}{m_b(m_b-2E_\gamma)}\right]\right\}$$

$$\left.\frac{d\Gamma_{88}^{ahc}}{dE_\gamma}\right|_{E_\gamma\to m_b/2} = \Gamma_{77}\cdot\frac{C_8^2}{9C_7^2}\frac{\alpha_s C_F}{4\pi}\frac{2}{m_b}\frac{\langle\bar B|\bar h(1-\gamma_5)\not{n}h|\bar B\rangle}{M_B}\times\left\{\frac{1}{\epsilon}-2-\gamma_E+\ln 4\pi-\ln\left[\frac{m_s^2}{\mu^2}\right]\right\},$$

Each diagram is divergent, but the sum is finite and equals to the previous calculation

Q_{8g}-Q_{8g}

- Interpretation:
  - The soft fields in the second diagram should not be contracted. Instead their diagram gives rise to the subleading shape function.
  - The contribution from the first diagram should be interpreted as part of the calculation of a subleading jet function.
  - After renormalization this equation will have the structure of a subleading jet function convoluted with the leading order shape function. (again, factorization work)



$$\int d\omega\, e^{-\frac{i}{2}\omega\bar{n}\cdot x} f_{88}(\omega) = \frac{1}{2M_B}\int_{-\infty}^{0} dy_+ \int_0^{\infty} dz_+$$

$$\Big(\langle\bar{B}|\bar{h}(0)\not{n}\not{\bar{n}}N^A(0,0)S_{\bar{n}}(0,0)\mathcal{Q}_{\bar{n}}(0,y_+)\bar{\mathcal{Q}}_{\bar{n}}(x_-,z_+)\not{\bar{n}}\not{n}S^{\dagger}_{\bar{n}}(x_-,0)N^A(x_-,x_-)h(x_-)|\bar{B}\rangle$$

$$-\langle\bar{B}|\bar{h}(0)\gamma_5\not{n}\not{\bar{n}}N^A(0,0)S_{\bar{n}}(0,0)\mathcal{Q}_{\bar{n}}(0,y_+)\bar{\mathcal{Q}}_{\bar{n}}(x_-,z_+)\not{\bar{n}}\not{n}\gamma_5 S^{\dagger}_{\bar{n}}(x_-,0)N^A(x_-,x_-)h(x_-)|\bar{B}\rangle$$

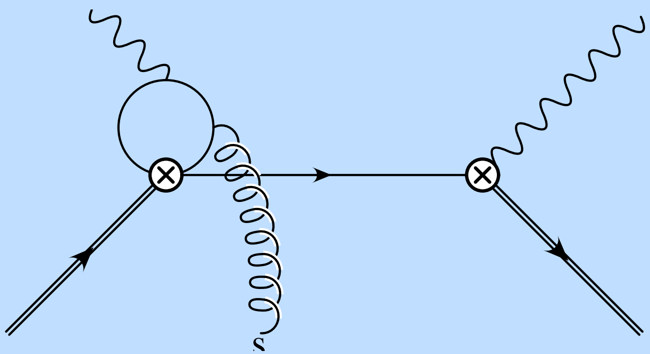
$$\langle\bar{B}|\bar{h}(0)\not{n}\not{\bar{n}}N^A(0,0)S_{\bar{n}}(0,0)\mathcal{Q}_{\bar{n}}(0,y_+)\bar{\mathcal{Q}}_{\bar{n}}(-x_-,z_+)\not{\bar{n}}\not{n}S^{\dagger}_{\bar{n}}(-x_-,0)N^A(-x_-,-x_-)h(-x_-)|\bar{B}\rangle^*$$

$$-\langle\bar{B}|\bar{h}(0)\gamma_5\not{n}\not{\bar{n}}N^A(0,0)S_{\bar{n}}(0,0)\mathcal{Q}_{\bar{n}}(0,y_+)\bar{\mathcal{Q}}_{\bar{n}}(x_-,z_+)\not{\bar{n}}\not{n}\gamma_5 S^{\dagger}_{\bar{n}}(-x_-,0)N^A(-x_-,-x_-)h(-x_-)|\bar{B}\rangle^*\Big)$$

$$\frac{d\Gamma_{88}}{dE_\gamma} = \frac{G_F^2\alpha|V_{tb}V_{ts}^*|^2 C_{7\gamma}^2 m_b^2}{2\pi^4}\left(\frac{E_\gamma m_b}{16}e_s^2\pi\alpha_s\frac{C_{8g}^2}{C_{7\gamma}^2}\int d\omega\,\delta(n\cdot p+\omega)f_{88}(\omega)\right)$$

Q_{1c}-$Q_{7\gamma}$

Non-local operators from 4-quark operator (from loop matching)



$$Q_1^c = \left(\frac{-ee_c}{4\pi^2}\right)\frac{1}{(q_\gamma + q_g)^2}\left[1 - F(r)\right] i(q_\gamma^\alpha + q_g^\alpha)\bar{\xi}\gamma^\beta(1-\gamma_5)gG_{\mu\alpha}h\,\epsilon^{\mu\beta\rho\sigma}F_{\rho\sigma}$$

$$F(r) = 4r\arctan^2\left(\frac{1}{\sqrt{4r-1}}\right) \qquad r = m_q^2/q^2,\; q^2 = (q_1+q_2)^2$$

Taking mc to be heavy, contribution to the total rate is given by

$$\Gamma_{71c} = -\Gamma_{77}\frac{C_1}{C_7\gamma}\frac{\lambda_2}{m_c^2}$$

[Voloshin
Ligeti, Randall, Wise
Grant, Morgan, Nussinov]

Taking mc to be hc:

WORK IN PROGRESS

Subleading shape function (1/mb suppression compared to LO)

Summary

- A complete basis of subleading shape function for $B \to X_s \gamma$ is analyzed using SCET: The effective weak Hamiltonian is matched onto SCET to NLO in $1/m_b$ and first order in the strong coupling constant g_s.

- The result is used to calculate the subleading shape functions originating from operators other than $Q_{7\gamma}$

- We have identified a new class of enhanced power corrections to the total inclusive decay rate from $Q_{7\gamma}$-Q_{8g}, which cannot be parameterized in terms of matrix elements of local operators. After the perturbative analysis of the decay rate being completed, the enhanced non-local power corrections will remain as the dominant source of theoretical uncertainty. (~5%)

- SCET reinterprets the previous calculations for Q_{8g}-Q_{8g}