DIS as $x \to 1$

DIS in moment space is a nice example of the OPE.

Take moments

$$M_N = \int_0^1 \mathrm{d}x \; x^{N-1} F(x, Q^2)$$

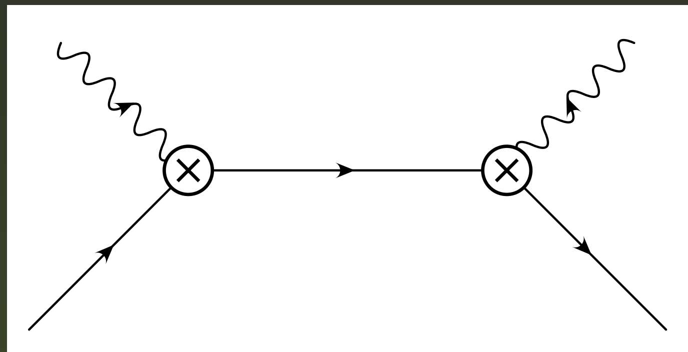
(use $\bar{N} = N e^{\gamma_E}$)

Scales:

Hard: Q^2 Jet: $\frac{Q^2}{\bar{N}}$ Soft: $\frac{Q^2}{\bar{N}^2}$ QCD: Λ^2_{QCD}

Jet: $Q^2(1-x)$

Generic x

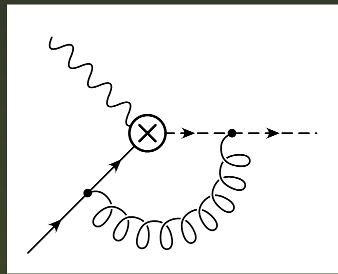


DIS at generic values of x: Do the OPE at Q, and match onto $C_N(Q)$ and operators $O_N(Q)$. The operators are moments of the parton distribution function

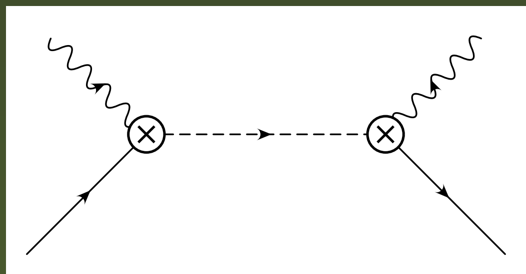
$$\bar{\psi}(x)\,\gamma^+\,W(x,0)\,\psi(0)$$

$x \to 1$

Break up the coefficient into the hard part at Q^2:



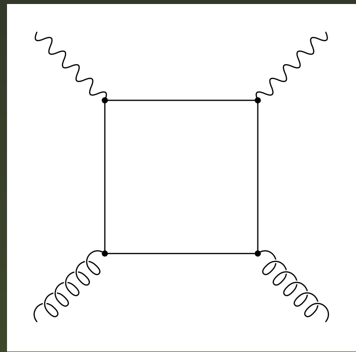
Jet part at $Q^2/\bar{N}$:



Left with the parton distribution at Λ_{QCD}.

Regions

Compute scattering off a parton:

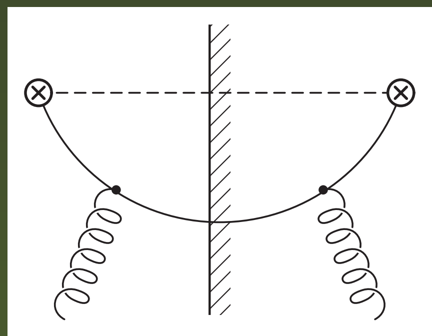


$$g_{1G} \quad = \quad h\frac{\alpha_s}{4\pi}\left[(2x-1)\ln\frac{Q^2(1-x)}{m^2x-p^2x^2(1-x)}\right.$$
$$\left. +3-4x+\frac{p^2x(1-x)}{m^2-p^2x(1-x)}\right].$$

$Q^2(1-x) \sim Q^2/\bar{N}$, $m^2 \sim \Lambda^2_{\mathrm{QCD}}$, and $p^2(1-x) \sim \Lambda^2_{\mathrm{QCD}}/\bar{N}$
[Messenger scale $p^2(1-x)$ needs off-shellness]

$$O_{\Delta q}(k^+) = \frac{1}{4\pi}\int \mathrm{d}z^- e^{-iz^-k^+}[\bar{\psi}(z^-)W(z^-)\gamma^+\gamma_5\psi(0)$$
$$+\bar{\psi}(0)W^\dagger(z^-)\gamma^+\gamma_5\psi(z^-)]$$

Using an off-shell gluon target gives



$$f_{\Delta q/G} = \frac{\alpha_s}{2\pi}\left[(2x-1)\ln\frac{\mu^2}{m^2-p^2x(1-x)} - 1 + \frac{m^2}{m^2-p^2x(1-x)}\right],$$

This gives:

$$\hat{g}_{1G} \quad = \quad \frac{\alpha_s}{4\pi}\left[(2x-1)\ln\frac{Q^2(1-x)}{\mu^2 x}+3-4x\right].$$

Depends on the jet scale $Q^2(1-x)$. All dependence on m^2 and $p^2(1-x)$ has dropped out.

EFT is used to compute perturbative quantities at Q^2 and $Q^2/\bar{N}$.

All scales Λ_{QCD} and below are included in the non-perturbative matrix element of the parton distribution function.

The parton diagram depends on m^2 and $p^2(1-x)$. One can take a well-defined parton distribution f that depends on Λ_{QCD}, and break it up into two parts based on a method of regions computation of a perturbation theory integral, $f = S \otimes M$.

The different pieces S and M can depend on the messenger scale, but the physical cross-section depends only on the parton distribution f which does not depend on the messenger scale.

In perturbation theory, with $m^2 = 0$ and $p^2 \neq 0$, one can have $q\bar{q}$ states with invariant mass $p^2(1-x)$. But if $m^2 \neq 0$, then the states are cutoff by $4m^2$. In QCD, there are no low-mass states.

$$m^2 - p^2 x(1-x)$$

Method of regions — use to determine which perturbative modes are integrated out, rather than which non-perturbative modes are retained.