Soft And Zero-Bin Subtractions

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Outline

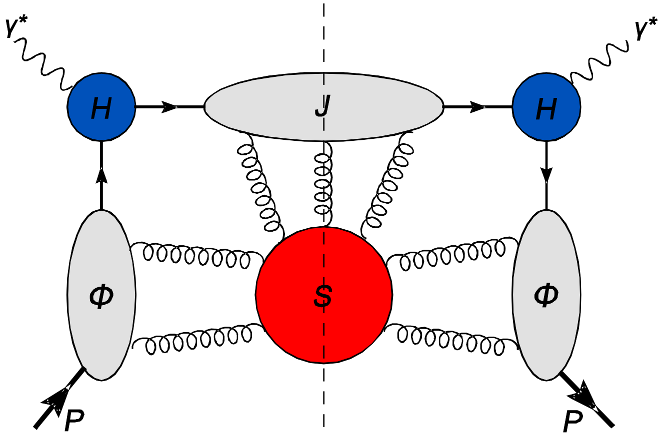
- Soft Subtraction In Perturbative QCD; Zero-Bin in SCET
- Quark Form Factor (QFF) In SCET: One Loop Results
- Zero-Bin For DIS In SCET
- Zero-Bin at Two-loop
- Remarks On ``An All Order Equivalence''?
- Conclusions

Soft Subtraction In pQCD

- Deep Inelastic Scattering At large-x:

$$F_{2,N} = H\,\phi_N\,J_N\,S_N$$

(Sterman,87)

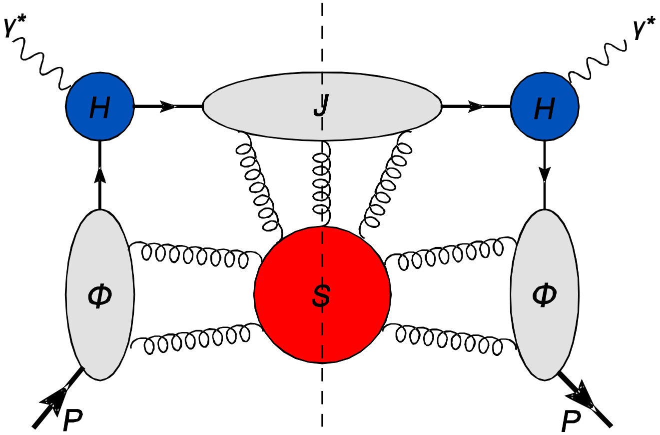


- The PDF And The Jet Function: Are Not Gauge Invariant
  [To Allow Resummation Of Large Logs]

Soft Subtraction: Continued

- For Gauge Invariant Quantities:

$$F_{2,N} = H \frac{\phi_N}{S_N} \frac{J_N}{S_N} S_N$$



- Subtracting The Soft Factor From The Collinear Matrix Elements: Avoid Double Counting

(Sterman et.al,98)

- Similarly: SIDIS and DIS (Ji et.al, 03 and 06); QFF (Collins, 89)

Zero-Bin

- For Certain EFT of QCD: NRQCD And SCET

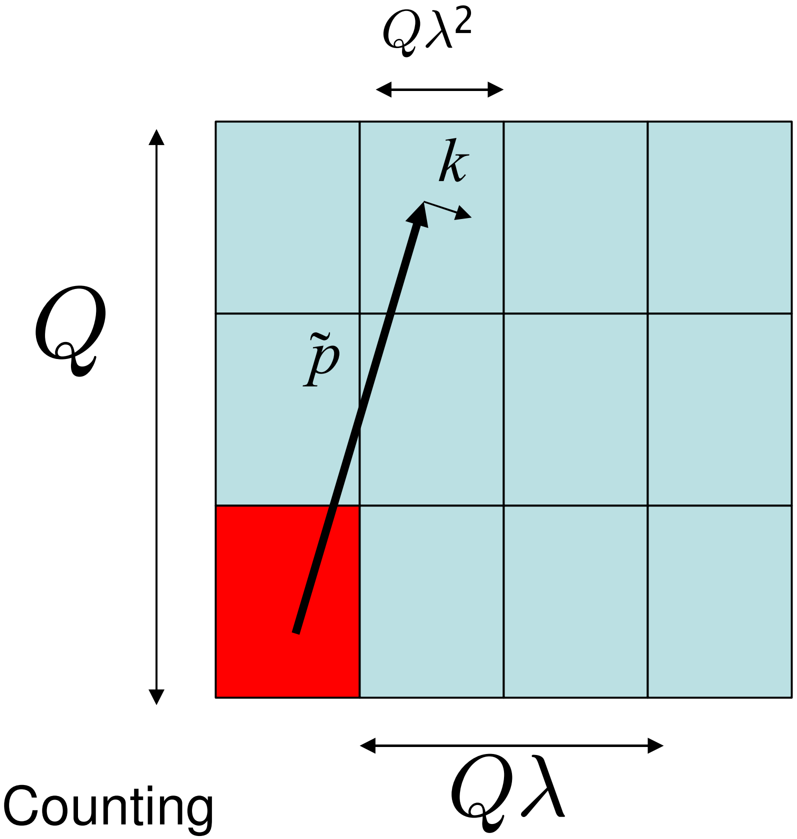
(Manohar and Stewart,06)

$$\phi(x) = \sum_{\tilde{p} \neq 0} e^{-i\,\tilde{p}\cdot x} \phi_{\tilde{p}}(x)$$

Collinear Scaling: $Q(1, \lambda^2, \lambda)$

(U)Soft Scaling: $Q(\lambda^2, \lambda^2, \lambda^2)$

$$\sum_{\tilde{p}=0} \int d^d k \rightarrow \int d^d l$$



- The Inclusion Of Zero-Bis: Leads To Double Counting

Few Remarks

- Soft Factor: Well-Defined Object At The Operator Level

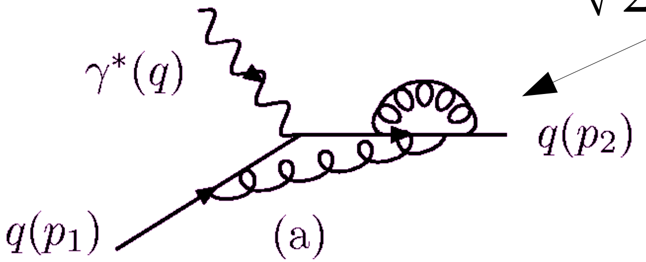
- The Notion of Zero-Bin Contribution is ``Diagrammatic'': No Closed Form!!

- The Equivalence Of The Two Prescriptions Is Not Trivial

- There Is A Need For (At Least) One Explicit Calculation

Quark Form Factor

$$\bar{n}=\frac{1}{\sqrt{2}}(1,0,0,-1)$$



$$n=\frac{1}{\sqrt{2}}(1,0,0,1)$$

- Full QCD

$$\langle j^\mu\rangle=\langle q(p_2)|\bar{\Psi}(x)\gamma^\mu\psi(x)|q(p_1)\rangle$$

- SCET Effective Current

$$\langle j^\mu\rangle_{eff}=C(Q^2)\gamma^\mu\langle 0|W_n^{(0)\dagger}\xi_n^{(0)}|q(p_1)\rangle\times\langle q(p_2)|\bar{\xi}_{\bar{n}}W_{\bar{n}}^{(0)}|0\rangle$$

$$\times\langle 0|Y_{\bar{n}}^\dagger Y_n|0\rangle$$

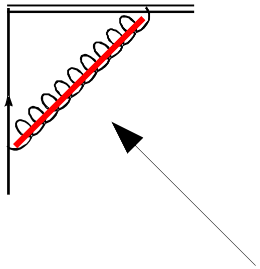
$$W_n(x)=P\exp\left[ig_s\int_{-\infty}^0 ds\,\bar{n}\cdot A_n(\bar{n}s+x)\right]$$

$$Y_n(x)=P\exp\left[ig_s\int_{-\infty}^0 ds\,n\cdot A_s(ns+x)\right]$$

One Loop Results: Pure DR

- For Incoming Jet $k^{+} \simeq Q$ $k^{-} \simeq Q \lambda^{2}$

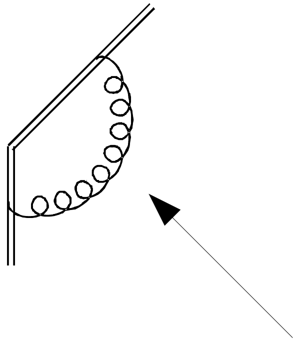
$$I_n = A \int_k \frac{\tilde{p} - k^{+}}{(k^{2} + i0)(-k^{-} + i0)(-2\, \tilde{p}\, k^{+} + k^{2} + i0)}$$



Collinear Gluon

$A = -2 \mathrm{i} g_s^{2} C_F (\mu^{2})^{\epsilon}$ $\tilde{p} = Q/\sqrt{2}$

$$I_s = \frac{A}{2} \int_k \frac{1}{(k^{2} + i0)(-k^{-} + i0)(-k^{+} + i0)}$$



Soft Gluon

- Zero-Bin Of I_n

$$I_{n,0} = I_s$$

QFF

- SCET:

$$I=(I_n-I_{n,0})+(I_{\bar{n}}-I_{\bar{n},0})+I_s$$
$$=I_n+I_{\bar{n}}-I_s$$

$$\langle j^\mu\rangle_{eff}=\gamma^\mu\left[1+\frac{\alpha_s}{4\pi}C_F\left(\frac{2}{\epsilon_{UV}^2}-\frac{2\ \ln r-3}{\epsilon_{UV}}\right)-\left(\frac{2}{\epsilon_{IR}^2}-\frac{2\ \ln r-3}{\epsilon_{IR}}\right)\right]$$

$$r=Q^2/\mu^2$$

- Full QCD:

$$\langle j^\mu\rangle=\gamma^\mu\left[1+\frac{\alpha_s}{4\pi}C_F\left(\frac{-2}{\epsilon_{IR}^2}+\frac{2\ \ln r-3}{\epsilon_{IR}}-\ln^2 r+3\ \ln r-8+\frac{\pi^2}{6}\right)\right]$$

- The Matching Coefficient And Anomalous Dimension: $C(r)$ And

$$\gamma_1=d\,C(r)/d\ \ln\mu$$

Suggestion!!

$$\langle 0|W_n^{\dagger}\xi_n|q\rangle_C=\frac{\langle 0|W_n^{\dagger}\xi_n|q\rangle}{\langle 0|Y_n^{\dagger}Y_{\overline{n}}|0\rangle}$$

- Does It Hold To Arbitrary Orders In PT?

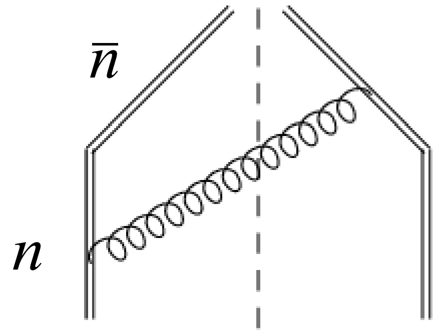
- Physical Observables (Cross Sections):

Emission Of Real Gluons

DIS At large-x

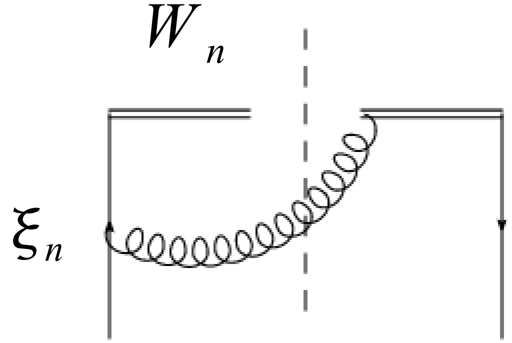
- Soft Factor

$$S(1-x)=\frac{\tilde{p}}{N_c}\int\frac{d\lambda}{2\pi}\mathrm{e}^{i\lambda(1-x)\,\tilde{p}}\langle 0|Tr[Y_n(\lambda n)Y^{\dagger}_{\bar{n}}(\lambda\,\bar{n})\times Y^{\dagger}_{\bar{n}}(0)\,Y_n(0)]|0\rangle$$



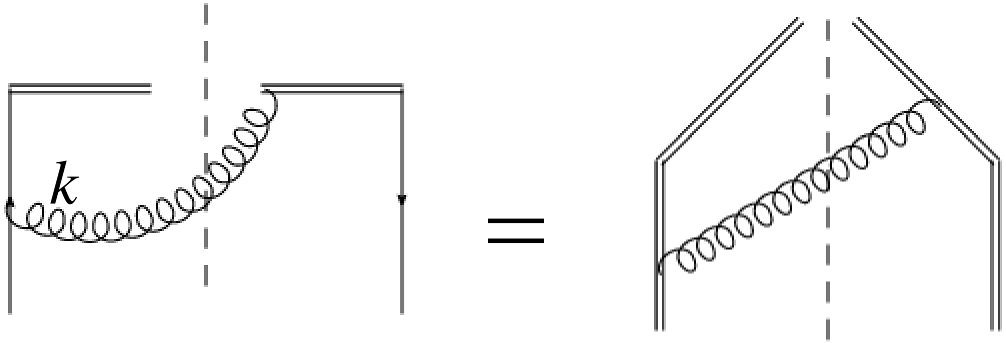
- PDF

$$\phi(x)=\frac{1}{2}\int\frac{d\lambda}{2\pi}\mathrm{e}^{i\lambda x\,\tilde{p}}\langle P|\,\bar{\xi}_n(\lambda\,\bar{n})W_n(\infty,0;,\lambda\,\bar{n})\times\gamma^{+}W^{\dagger}_n(\infty,0;0)\,\xi_n|P\rangle$$



Results: Soft And PDF

- At Large-x



$$I_s = \frac{\alpha_s}{4\pi} C_F \left[\frac{-1}{\epsilon_{IR}} \delta(1-x) + D_0(x) \right] \left(\frac{2}{\epsilon_{UV}} - \frac{2}{\epsilon_{IR}} \right)$$

$$D_i(x) = \left[\frac{\ln^i(1-x)}{(1-x)} \right]_+$$

- Reason: PDF Is Proportional To

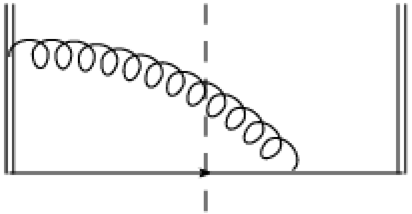
$$\delta(k^2)\,\delta(k^+ - (1-x)\,\tilde{p})$$

k Is Soft Not Collinear

Jet Function

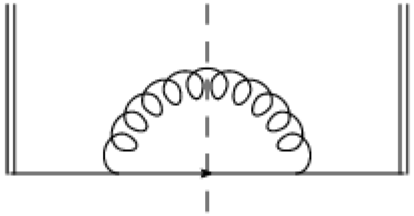
$$\langle 0|T\,[W_{n}^{\dagger}(z)\,\xi_{\bar{n}}(z)\;\bar{\xi}_{\bar{n}}(0)\,W_{\bar{n}}(0)]|0\rangle = i\,\frac{\bar{n}}{\sqrt{2}}\int_{k} \mathrm{e}^{-ikz}\;\tilde{J}(k)$$

(a)



$$J(Q^{2}\,,x)=\frac{-1}{\pi}\;\tilde{p}\times\Im[\,\tilde{J}\,]$$

(b)

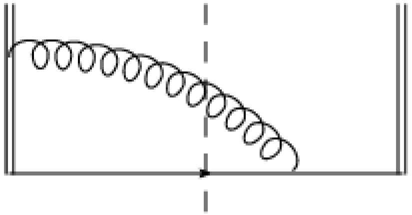


$$r=Q^{2}/\mu^{2}$$

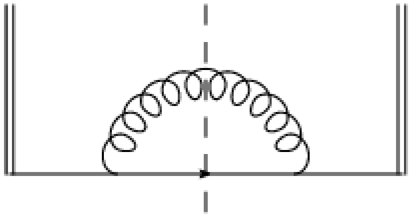
$$j\,(Q^{2}\,,x)=\frac{\alpha_{s}}{4\pi}C_{F}\left[-\frac{4\mathrm{D}_{0}(x)}{\epsilon_{IR}}+4\mathrm{D}_{1}(\,x)+4\,\ln r\,D_{0}(\,x)+\left(2\,\ln^{2}r-3\,\ln r+7-\pi^{2}\right)\delta(1-x)\right]$$
$$+\frac{\alpha_{s}}{4\pi}(r)^{-\epsilon}\left[\frac{4}{\epsilon_{UV}\,\epsilon_{IR}}+\frac{3}{\epsilon_{UV}}\right]\delta(1-x)$$

Jet Function-Continued

(a)



(b)



- The Zero-Bin Of (a) Has Exactly The Same Pole Structure As I_s

$$I_s = \frac{\alpha_s}{4\pi} C_F (r)^{-\epsilon} \left[-\frac{1}{\epsilon_{IR}} \delta(1-x) + D_0(x) \right] \left(\frac{2}{\epsilon_{UV}} - \frac{2}{\epsilon_{IR}} \right)$$

- The Diagram (b) Has No Zero-Bin

- The Collinear Jet

$$J(Q^2, x)_C = \frac{\alpha_s}{4\pi} \left[-\frac{D_0(x)}{\epsilon_{UV}} + 4\mathrm{D}_1(x) + 4 \ln r \, D_0(x) + \left(2 \ln^2 r - 3 \ln r + 7 - \pi^2 \right) \delta(1-x) \right]$$
$$+ \frac{\alpha_s}{4\pi} (r)^{-\epsilon} \left[\frac{4}{\epsilon_{UV}^2} + \frac{3}{\epsilon_{UV}} \right] \delta(1-x)$$

Renormalized Jet Function

$$J(Q^2, x)_C = \frac{\alpha_s}{4\pi}\left[-\frac{D_0(x)}{\epsilon_{UV}} + 4\mathrm{D}_1(x) + 4\ \ln r\, D_0(x) + \left(2\ \ln^2 r - 3\ \ln r + 7 - \pi^2\right)\delta(1-x)\right]$$

$$+\frac{\alpha_s}{4\pi}(r)^{-\epsilon}\left[\frac{4}{\epsilon_{UV}^2} + \frac{3}{\epsilon_{UV}}\right]\delta(1-x)$$

- In Moment Space: The Finite Jet Is The Matching Coefficient At $\mu_I = Q\sqrt{(1-x)}$

$$J_{c,N} = 1 + \frac{\alpha_s}{4\pi}\left[2\ \ln^2 w - 3\ \ln w + 7 - \frac{2}{3}\pi^2\right] \qquad w = \frac{Q^2}{\overline{N}\mu^2}$$

Putting Things Together

- To First Order In α_s : In Terms Of The Naive PDF And Jet

$$F_{2,N} = H \frac{J_N}{S_N} \frac{\phi_N}{S_N} S_N = H(r) \left(\frac{J}{S} \right)_N (w) \times \phi_N$$

$$r = Q^2 / \mu^2$$

$$w = \frac{Q^2}{\overline{N} \mu^2}$$

$$F_{2,N} = 1 + \frac{\alpha_s}{4\pi} C_F \left[- \frac{1}{\epsilon_{IR}} [3 - 4 \ln \overline{N}] - 2 \ln^2 r + 6 \ln r + 2 \ln^2 w - 3 \ln W - 9 - \frac{\pi^2}{3} \right]$$

$$\overline{N} = N \, \mathrm{e}^{\gamma_E}$$

An All Order Equivalence?

$$\langle 0|W_n^\dagger \xi_n|q\rangle_C = \frac{\langle 0|W_n^\dagger \xi_n|q\rangle}{\langle 0|Y_{\bar{n}}^\dagger Y_n|0\rangle}$$

- Could be Obtained To All Orders In PT Following Lee-Sterman Argument

(C. Lee And G. Sterman,06)

- It Is Useful To Perform Explicit Two-Loop Calculation

Two-Loop Results

$$\langle 0|W_n^\dagger \xi_n|q\rangle_C = \frac{\langle 0|W_{\bar{n}}^\dagger \xi_n|q\rangle}{\langle 0|Y_n^\dagger Y_{\bar{n}}|0\rangle}$$

$$\langle 0|W_n^\dagger \xi_n|q\rangle = \sum \alpha_s^k W^{(k)} \qquad \langle 0|Y_n^\dagger Y_n|0\rangle = \sum \alpha_s^k Y^{(k)}$$

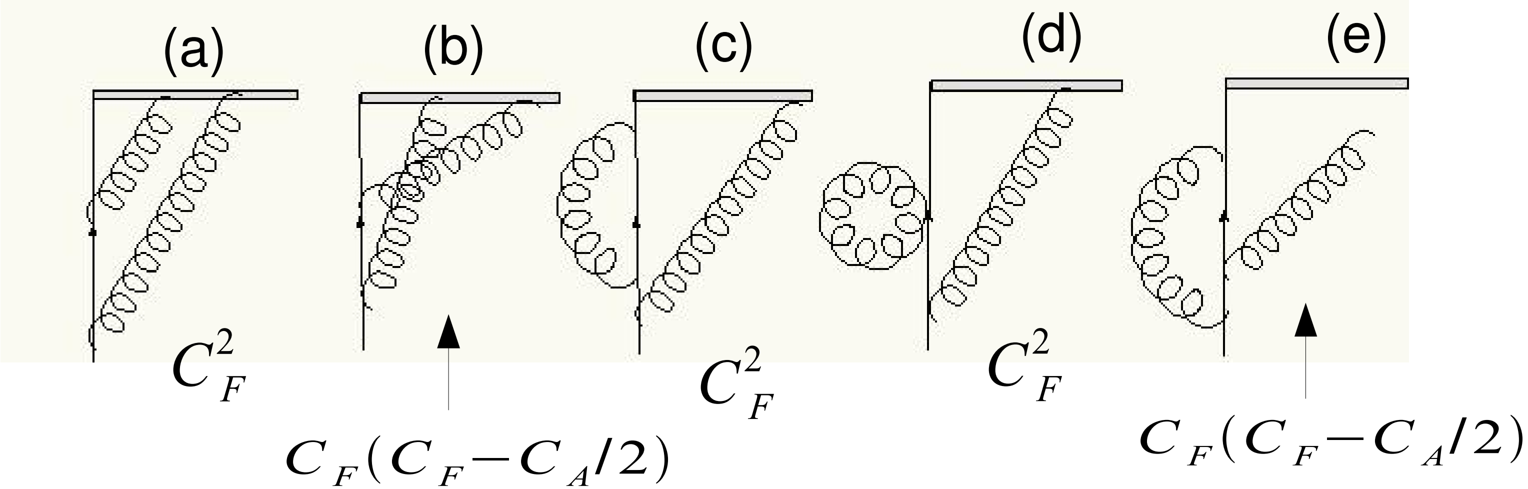
$$\langle 0|W_n^\dagger \xi_n|q\rangle_c = 1 + \alpha_s\left(W^{(1)} - Y^{(1)}\right) + \alpha_s^2\left(W^{(2)} - \underline{W^{(1)}Y^{(1)} - Y^{(2)} + \left(Y^{(1)}\right)^2}\right)$$

$\uparrow$

The Total Zero-Bin Contribution to

$W^{(2)}$

Abelian Contribtuion

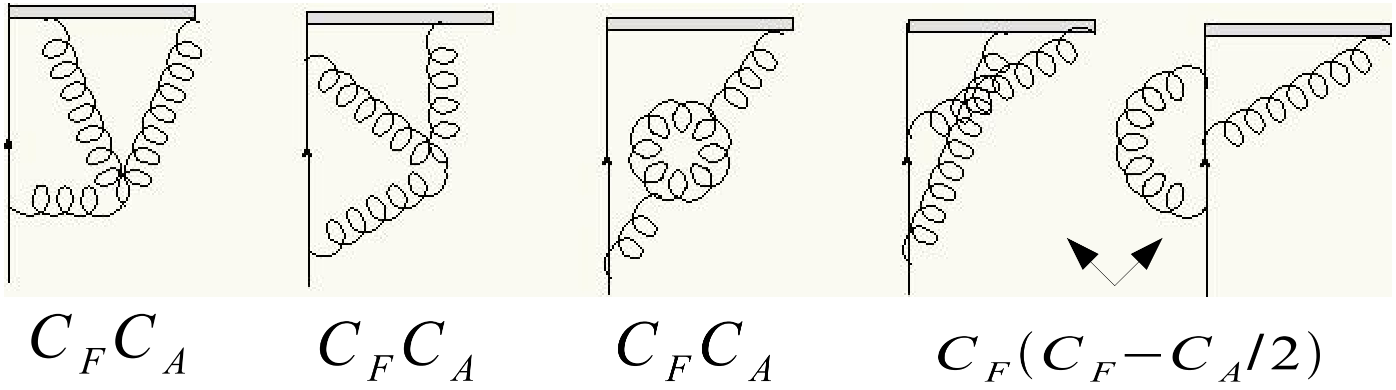


- All The Zero-Bin Contributions With C_F^2 From (a) And (b) Equal $[W^{(1)}Y^{(1)}+Y^{(2)}-(Y^{(1)})^2]_{C_F^2}$

$$\langle 0|W_n^\dagger \xi_n|q\rangle_c=1+\alpha_s(W^{(1)}-Y^{(1)})+\alpha_s^2(W^{(2)}-W^{(1)}Y^{(1)}-Y^{(2)}+(Y^{(1)})^2)$$

- For (c), (d) And (e) There Are Leading Zero-Bin Contributions When One Loop Momentum Is Soft And One Is Collinear. They Add Up To Zero. Great!!!

Non-Abelian Contribution



- Each One Has Leading Zero-Bin: One Soft And One Collinear

$$\langle 0|W_n^\dagger \xi_n|q\rangle_c = 1 + \alpha_s(W^{(1)} - Y^{(1)}) + \alpha_s^2(W^{(2)} - \underset{\overset{\uparrow}{C_F^2}}{W^{(1)}Y^{(1)}} - Y^{(2)} + \underset{\overset{\uparrow}{C_F^2}}{(Y^{(1)})^2})$$

$Y^{(2)}$: Contains Only Soft Gluons

- This Zero-Bin Contribution, If Not Zero, It Cannot Be Accounted For By Soft Factor Subtraction!!

- An Intricate Cancellation Among Zero-Bin Contributions From Five FD!!

Lee-Sterman Argument

- Include The Zero-Bin Gluons And Then Remove Them By ``Soft'' Field Redefinition

$$\langle 0|W_n^\dagger \xi_n|q\rangle \Rightarrow \langle 0|Y_n^\dagger W_n^\dagger \xi_n|q\rangle$$

- Make Another Field Redefinition To Decouple Soft Gluons In The Conjugate Direction

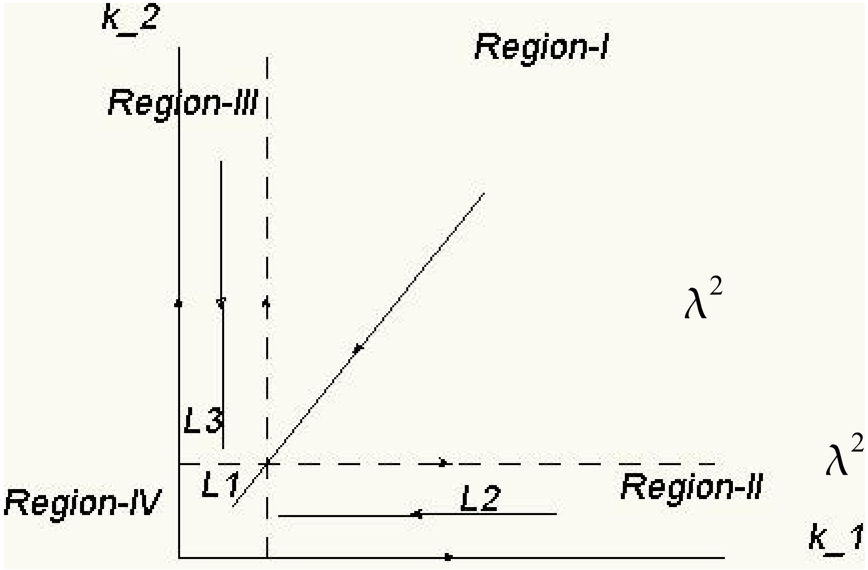
$$W_n \Rightarrow W_n Y_{\bar{n}}^\dagger$$

$$\langle 0|W_n^\dagger \xi_n|q\rangle = \langle 0|Y_n^\dagger Y_{\bar{n}}|0\rangle \langle 0|W_n^\dagger \xi_n|q\rangle_c$$

- How Do We See The Cancellation Of One Soft + One Collinear For The Two-Loop Case??

Final Statement

There Is Much More To Be Understood On The Equivalence Of Soft And Zero-Bin Subtractions. This Applies For Inclusive, Semi-Inclusive And Exclusive Processes (Form Factors). [Work In Progress]


k_2
Region-I
Region-III
λ^2
L3
λ^2
L1
Region-IV
L2
Region-II
k_1