Transverse Momentum Dependent Parton Distribution Functions in SCET

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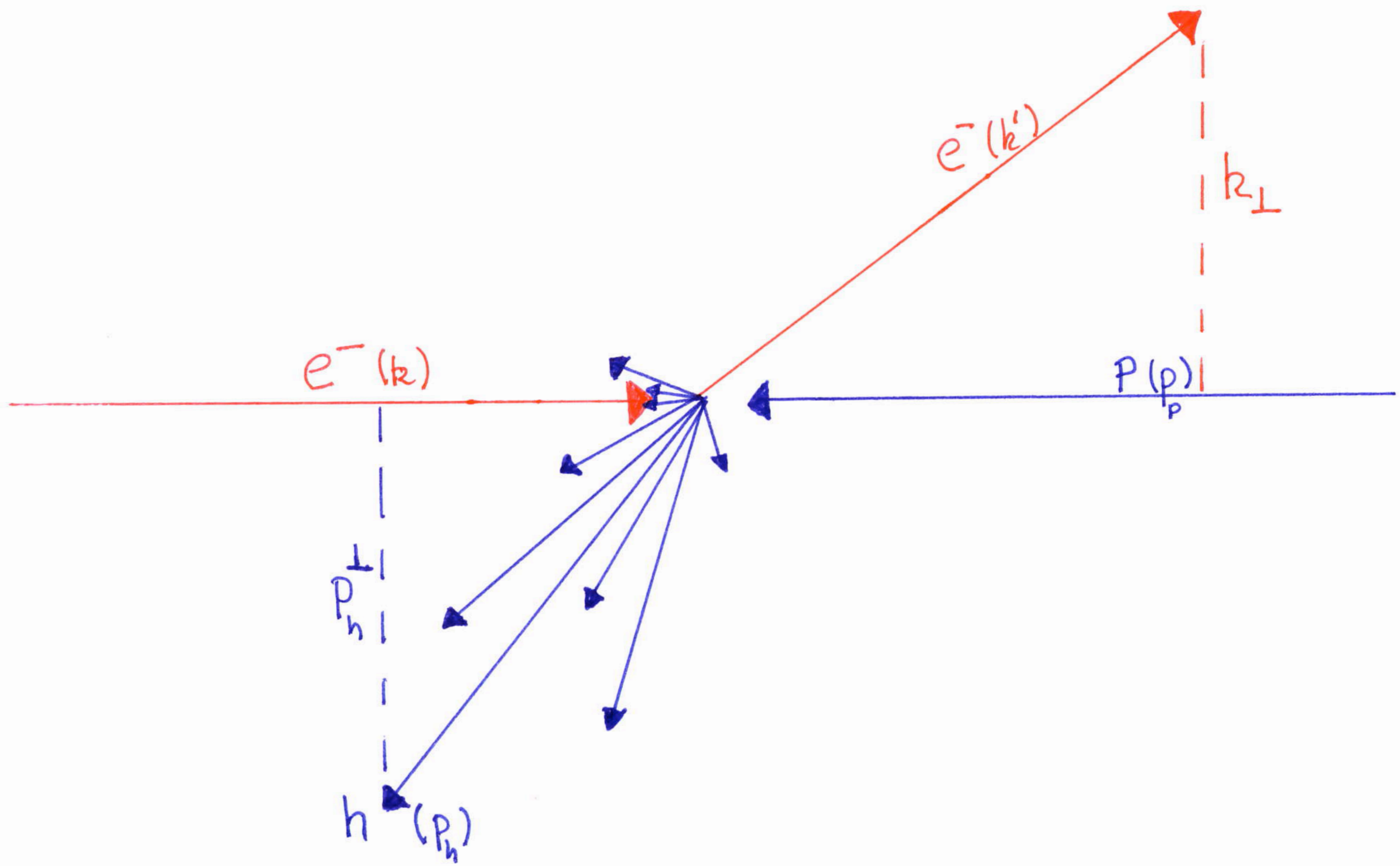
$$h_v(x) = e^{i m \not{v}\, v_\mu x^\mu} \psi(x) \tag{4}$$

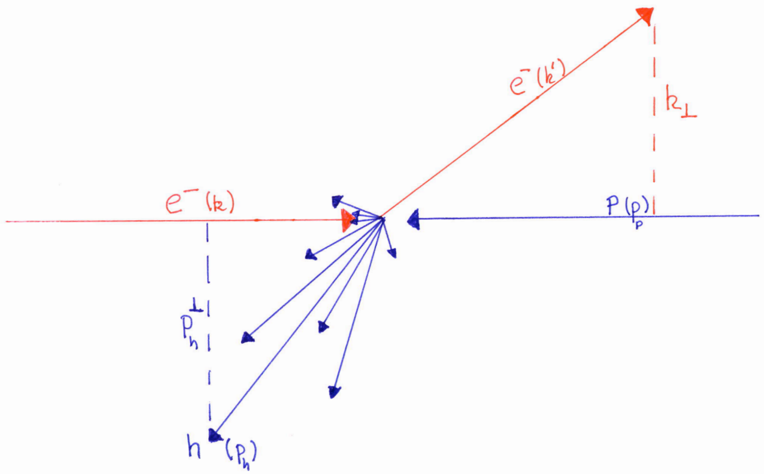
In the full QCD theory, it would be crazy to redefine the field in this way, because of the explicit dependence on the velocity, v^μ.

Howard Georgi

Semi-Inclusive DIS (SIDIS)



SIDIS



- Large momentum transfer:

$$-Q^2 \equiv q^2 \equiv (k' - k)^2 \gg \Lambda^2_{\rm QCD}$$

- Observed hadron (h) in the final state, but otherwise inclusive
- Variables: $x_B = \dfrac{Q^2}{2p_p \cdot q}$ $\quad z_h = \dfrac{p_h \cdot p_p}{q \cdot p_p}$ $\quad y = \dfrac{p_p \cdot q}{p_p \cdot k}$ $\quad q_T$
- Arbitrary transverse momentum q_T
- Angular averaged

SIDIS

- Restrict ourselves to q_T on the order of the hadronic scale $q_T \sim \Lambda$
  - Probes non-perturbative structure in the transverse direction

$\Longrightarrow$ Transverse Momentum Dependent Factorization

Meng, Olness, Soper, Phys. Rev. D54, 1919 (1996)

Ji, Ma, Yuan, Phys. Rev. D 71, 034005 (2005)

Idilbi, Ji, Ma, Yuan, Phys. Rev. D 73, 077501 (2006)

SIDIS

- Angular averaged cross section

$$d\sigma = \frac{2\pi\alpha^2}{Q^4}\mathcal{L}_{\mu\nu}W^{\mu\nu}$$

Lepton tensor

$$\mathcal{L}_{\mu\nu} = -Q^2\frac{1+(1-y)^2}{y}g^{\perp}_{\mu\nu}$$

Hadronic tensor

$$W^{\mu\nu} = \frac{1}{4z_h}\sum_X\int\frac{d^4x}{(2\pi)^4}e^{iq\cdot x}\langle p_p|J^{\mu\dagger}(x)|Xp_h\rangle\langle Xp_h|J^{\nu}(0)|p_p\rangle$$

SCET in SIDIS

- In the Breit frame: $q^\mu \approx \frac{Q}{2}(\bar{n}^\mu - n^\mu) + q_\perp^\mu$

  With $n^\mu = (1,0,0,-1) \quad \bar{n}^\mu = (1,0,0,1)$
- In the region where, $q_\perp \sim \Lambda \sim Q\eta\,, \quad \eta \sim \frac{\Lambda}{Q}$

$\Longrightarrow$ SCET_{II}

- First match onto the QCD current onto an SCET_{I} current

$$J^\mu = \bar{\psi}(x)\gamma^\mu\psi(x) \to \sum_{\omega\bar{\omega}} C(\omega,\bar{\omega})\overline{\chi}_{\bar{n}\bar{\omega}}(x)\gamma_\perp^\mu\chi_{n\omega}(x)$$

Hadronic Tensor in $\mathrm{SCET_I}$

- Factor the cross section

$$W^{\mu\nu}_{\mathrm{eff}} = \frac{-g_{\perp}^{\mu\nu}}{8z_h}|C(Q)|^2 \int \frac{d^4x}{(2\pi)^4} e^{iq_r\cdot x} \langle p_n|\overline{\chi}_{nQ}(x)\frac{\bar{\not{n}}}{2}\chi_{nQ}(0)|p_n\rangle$$

$$\times \sum_{X_{\bar{n}}} \langle 0|\frac{\not{n}}{2}\chi_{\bar{n}Q}(x)|X_{\bar{n}}p_{\bar{n}}\rangle\langle X_{\bar{n}}p_{\bar{n}}|\overline{\chi}_{\bar{n}Q}(0)|0\rangle$$

$$\times \langle 0|Y_n^{\dagger}(x)Y_{\bar{n}}(x)Y_{\bar{n}}^{\dagger}(0)Y_n(0)|0\rangle$$

- q_r : residual momentum

Hadronic Tensor in $\mathrm{SCET}_{\mathrm{II}}$

- Match onto $\mathrm{SCET}_{\mathrm{II}}$

$$W^{\mu\nu}_{\mathrm{eff}} = \frac{-g_\perp^{\mu\nu}}{16 z_h} |C(Q)|^2 \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d^2 \ell_\perp}{(2\pi)^2} S(q_\perp + k_\perp + \ell_\perp)$$

$$\times f_{i/p}(x_B, k_\perp) F_{h/i}(z_h, \ell_\perp)$$

Agrees with Ji, Ma, Yuan, Phys. Rev. D 71, 034005 (2005)

- Overlapping regions are subtracted
- Soft function: $S(q_\perp)$
- TMD parton distribution function: $f_{i/p}(x_B, k_\perp)$
- TMD fragmentation function: $F_{h/i}(z_h, \ell_\perp)$

Transverse Momentum Dependent Parton Distribution Function

- Operator Definition (?) of the TMD pdf

$$\frac{1}{2}\sum_{\text{spin}}\langle p_p|\bar{\xi}_n W_n(x)\delta(\bar{\mathcal{P}}_+ - \omega_+)\bar{n\!\!\!/} W_n^\dagger \xi(0)|p_p\rangle =$$

$$= 2\delta(x^+)\int \frac{dk^+}{2\pi}\frac{d^2k_\perp}{(2\pi)^2}e^{-\frac{i}{2}k^+x^-}e^{-ik_\perp\cdot x_\perp}F_{i/p}\left(\frac{\omega_+}{2\bar{n}\cdot p_p}, k_\perp\right)$$

- Implicit subtraction of overlapping regions

TMD pdf at one loop

- Regularization:
  - Dim. Reg. for UV
  - Offshellness ($p^2 \neq 0$) for IR
- Unregulated UV divergence in “real emission” contribution

$$\propto \frac{1}{k_\perp^2} \int^\infty \frac{dx^-}{x^-}$$

- Divergence is due to infinite rapidity!
  - Improperly defined operator

Collins, Acta Phys. Polon. B 34 (2003) 3103 [hep-ph/0304122]

TMD pdf operator with a Rapidity Regulator

- "Since there are divergences in the pdf, as we have seen, the definition must be modified to incorporate some kind of cuttoff in gluon rapidity or some kind of general renormalization."

Collins, Acta Phys. Polon. B 34 (2003) 3103 [hep-ph/0304122]

$$1)\sum_{\text{spin}} \langle p_p | \bar{\psi} W_v^\dagger(x^-, x_\perp) \not{n} W_v \psi(0) | p_p \rangle$$

Ji, Ma, Yuan, Phys. Rev. D 71, 034005 (2005)

$v = (v_+, v_-, 0_\perp)$ is off the lightcone.

$$2)\ \sum_{\text{spin}} \frac{\langle p_p | \bar{\psi} W_n^\dagger(x^-, x_\perp) \not{n} W_n \psi(0) | p_p \rangle}{\langle 0 | Y_v(x^-, x_\perp) Y_n^\dagger(x^-, x_\perp) Y_n(0) Y_v^\dagger(0) | 0 \rangle / \langle 0 | Y_v(x^-) Y_n^\dagger(x^-) Y_n(0) Y_v^\dagger(0) | 0 \rangle}$$

Collins and Hautmann, Phys. Lett. B 472 (2000) 129; JHEP 0103 (2001) 016

Hautmann [hep-ph/0702196]

TMD pdf operator with Rapidity Regulator in Dim. Reg.

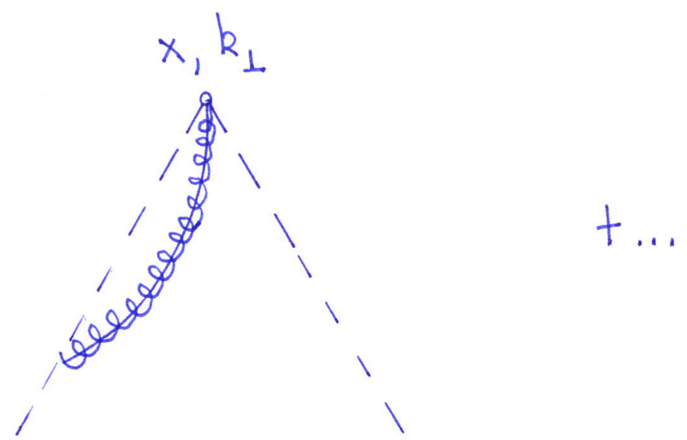
A. V. Manohar and I. W. Stewart, hep-ph/0605001

$$\sum_{\text{spin}} \langle p_p | \bar{\xi}_n W_n(x) \delta(\bar{\mathcal{P}}_+ - \omega_+) \bar{n}\!\!\!/ W_n^\dagger \xi(0) | p_p \rangle \to$$

$$\sum_{\text{spin}} \langle p_p | \bar{\xi}_n W_n(x) \left| \frac{\bar{\mathcal{P}}^\dagger}{\mu_-} \right|^\epsilon \delta(\bar{\mathcal{P}}_+ - \omega_+) \bar{n}\!\!\!/ \left| \frac{\bar{\mathcal{P}}}{\mu_-} \right|^\epsilon W_n^\dagger \xi(0) | p_p \rangle$$

- Required in SCET_{II} to maintain factorization of soft and collinear

TMD pdf at one loop

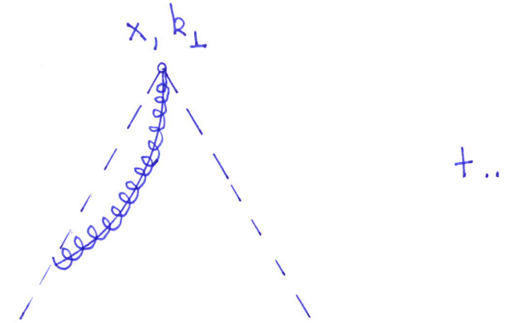


- "Real emission" is regulated in the UV

$$\frac{\alpha_s C_f}{2\pi^2\epsilon}\delta(1-x_B)\int d^2k_\perp\, e^{ib_\perp\cdot k_\perp}\frac{1}{k_\perp^2+p^2} = \frac{\alpha_s C_f}{\pi\epsilon}\delta(1-x_B)\ln\frac{4}{b_\perp^2 p^2}$$

Fourier Transform into impact parameter space

TMD pdf at one loop



- "virtual" contribution: Dim. reg. regulates UV and offshellness regulates the IR

$$-\frac{\alpha_s C_f}{\pi \epsilon^2}\left(\frac{x^2 p_p^{-2}\mu^2}{p^2 \mu_-^2}\right)^{\epsilon} \delta(1-x_B)\delta^2(k_\perp) \approx -\frac{\alpha_s C_f}{\pi}\delta(1-x_B)\left[\frac{1}{\epsilon^2}+\frac{1}{\epsilon}\ln\left(\frac{x^2 p_p^{-2}\mu^2}{p^2\mu_-^2}\right)\right]$$

- Real + virtual

$$-\frac{\alpha_s C_f}{\pi}\delta(1-x_B)\left[\frac{1}{\epsilon^2}+\frac{1}{\epsilon}\ln\left(\frac{x_B^2 b_\perp^2 p_p^{-2}\mu^2}{4}\right)\right]$$

Conclusions

- Rapidity factorization in $SCET_{II}$ regulates "real emission"
- Regulate all UV divergences using Dim. Reg.
- No need for a counter-term operator
- What's left to do:
    - Does it reproduce Collins--Soper evolution?
    - Soft function: one loop and running
    - Fragmentation function: one loop and running
    - IR divergences reproduce QCD?!?!?!