29 MAR 2007 SCET Workshop

Event Shapes and SCET

SCET05, 06

I Review: factorization of $\frac{d\sigma}{de}$ in SCET — CB, CL, AM, MW

II Comparison: factorization in QCD — CL, GS

III Update: — CB, SF, CL, GS

A better way to factorize $\frac{d\sigma}{de}$ $\cancel{|N\rangle = |J_1 J_2 X_S\rangle}$

I Factorizing event shape distributions in SCET

$$e(N) = \frac{1}{Q}\sum_{i\in N} |\vec{p}_i^{\perp}|\, e^{-\eta_i} f_e(\eta_i)$$

$$\tau = \frac{1}{Q}\sum_i |\vec{p}_i^{\perp}|\, e^{-|\eta_i|} = 1 - \frac{1}{Q}\sum_i |\hat{t}\cdot\vec{p}_i|$$

$$C = \frac{3}{2Q^2}\sum_i |\vec{p}_i||\vec{p}_j| \sin^2\theta_{ij} = \frac{1}{Q}\sum_i |\vec{p}_i^{\perp}| \frac{3}{\cosh\eta}$$

$$\tau_a = \frac{1}{Q}\sum_i |\vec{p}_i^{\perp}|\, e^{-|\eta_i|(1-a)} = \frac{1}{Q}\sum_i E_i \sin^a\theta_i\,(1-\cos\theta_i)^{1-a}$$

pQCD ⇒ $\langle e\rangle = \langle e\rangle_{PT} + \frac{C_e A}{Q}$

1-gluon emission

insert sec. III

$$\frac{d\sigma}{de} = \frac{1}{2Q^2}\sum_N |\langle N|j_2^\mu|0\rangle L_\mu|^2 (2\pi)^4\delta^4(Q-p_N)\,\delta(e-e(N))$$

$$j_2^\mu = \bar{\chi}_n Y_n \gamma_\perp^\mu \bar{Y}_{\bar{n}} \chi_{\bar{n}}$$

$$= \int de_J \frac{1}{2Q^2}\sum_{J_n J_{\bar{n}}} |\langle J_n J_{\bar{n}}|\bar{\chi}_n \gamma_\perp^\mu \chi_{\bar{n}}|0\rangle L_\mu|^2\, \delta(e_J - e(J_1 J_2))$$

$$\times \frac{1}{N_c}\mathrm{Tr}\sum_{X_S} |\langle X_S|Y_n \bar{Y}_{\bar{n}}|0\rangle|^2\, \delta(e-e_J-e(X_S))\,(2\pi)^4\delta^4(Q-p_{J_n}-p_{J_{\bar{n}}}-p_{X_S})$$

$$= \int de_J\, \sigma_J(e_J)\,\delta(e-e_J)$$

$$\langle e\rangle = \langle e\rangle_{PT} + \frac{1}{N_c}\mathrm{Tr}\sum_{X_S}\langle 0|\bar{Y}_{\bar{n}}^\dagger Y_n^\dagger|X_S\rangle\langle X_S|Y_n\bar{Y}_{\bar{n}}|0\rangle\, e(X_S)$$

$$e(X_S)|X_S\rangle = \frac{1}{Q}\sum_{i\in N}|\vec{k}_i^{\perp}|\, f_e(\eta_i)|X_S\rangle = \frac{1}{Q}\int d\eta\, \mathcal{E}_T(\eta)\, f_e(\eta)\,|X_S\rangle$$

$$\mathcal{E}_T(\eta)|X\rangle = \sum_{i\in N}|\vec{k}_i^{\perp}|\,\delta(\eta-\eta_i)\,|X\rangle$$

$$\cancel{\langle e\rangle = \langle e\rangle_{PT} + \frac{1}{Q}\frac{1}{N_c}\mathrm{Tr}\langle 0|\bar{Y}_{\bar n}^{\dagger} Y_n^{\dagger}\int d\eta\, f_e(}$$

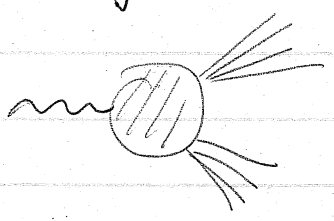
$$\langle e\rangle = \langle e\rangle_{PT} + \frac{1}{Q}\underbrace{\int d\eta\, f_e(\eta)}_{c_e}\ \frac{1}{N_c}\mathrm{Tr}\langle 0|\bar{Y}_{\bar n}^{\dagger} Y_n\, \mathcal{E}_T(\eta)\, Y_n \bar{Y}_{\bar n}|0\rangle$$

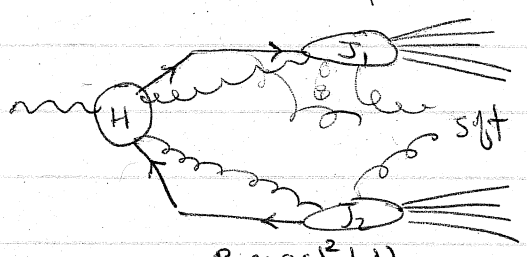
$U^{-1}U$

$$\langle 0|\bar{Y}_{\bar n}^{\dagger} Y_n\, \mathcal{E}_T(\eta')\, Y_n \bar{Y}_{\bar n}|0\rangle$$

$$= \langle e\rangle_{PT} + \frac{1}{Q} c_e \mathcal{A}$$

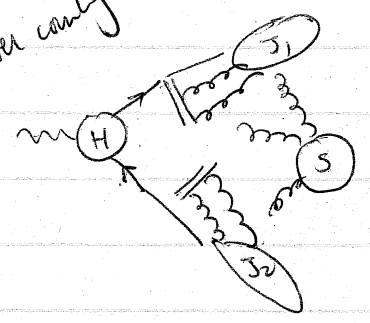
II Event shape factorization in QCD — Beyer Kucs Sterman (03)





$p_J \sim Q(\lambda^2, 1, \lambda)$
$Q(1, \lambda^2, \lambda)$

power counting



same rules as leading order SCET

Sum of diagrams ~~coupling~~ linking short-distance function gluons to jets ; $h(p_{J_1}, p_{J_2}, \mathcal{A}^{q\bar q}) \xrightarrow{\text{sum}} \Phi^{\bar q}_{\xi_2}(0,-\infty)\, h_2(p_{J_c}, \xi_c)\, \Phi^{q}_{\xi_1}$

individual gluon momenta in diagrams

we choose $\xi_{1,2} = \bar n_1, \bar n_2$

$$\frac{d\sigma}{de} = \sigma_0(Q)\int de_n\, de_{\bar n}\, de_s\, \delta(e - e_n - e_{\bar n} - e_s)\, J_n(e_n)\, J_{\bar n}(e_{\bar n})\, S(e_s)$$

$\sigma_0(Q) \to |h_2|^2$

$$J_c \sim \sum_{N_{J_c}} \mathrm{Tr}\left[\gamma^{\mu}\langle 0|\Phi^{(q)\dagger}_{\xi_c}(0)\, q(0)|N_{J_c}\rangle\langle N_{J_c}|\bar q(0)\, \Phi^{(q)}_{\xi_c}(0)|0\rangle\right]\delta(\ \ )$$

include soft partons (IR safety)

N_J includes soft but collinear partons

$\Rightarrow$ double-counting

construction of the soft function:

power counting: soft-to-collinear coupling same as soft-to-eikonal line coupling

"eik and" cross-section

$$\bar{\sigma}_{eik}(e) = \frac{1}{N_c} \sum_{N_{eik}} \langle 0 | \Phi^{\bar{q}\,\dagger}_{n_2}(\infty,0)\, \Phi^{q\,\dagger}_{n_1}(\infty,0) | N_{eik}\rangle\langle N_{eik} | \Phi^{q}_{n_1}(\infty,0)\, \Phi^{\bar{q}}_{n_2}(\infty,0) | 0\rangle \times \delta(e - e(N_{eik}))$$

- modulo soft radiation away from jets
- contains coll radiation $\Rightarrow$ factor out

$$\bar{\sigma}_{eik}(e) = \int de_s\, de_1\, de_2\; J_1^{eik}(e_1)\, J_2^{eik}(e_2)\, S(e_s)\; \delta(e - e_1 - e_2 - e_s)$$

$$J_1^{eik} = \sum_{N_1^{eik}} \langle 0 | \Phi^{\dagger}_{\xi_1}(0,-\infty)\, \Phi^{\dagger}_{n_1}(\infty,0) | N_1^{eik}\rangle\langle N_1^{eik} | \Phi_{n_1}(\infty,0)\, \Phi_{\xi_1}(0,-\infty) | 0\rangle$$

replace gluons with eikonal lines emitting soft gluons

Same soft function as full cross-section

$$\int de_s\, f(e)\, g(e - e_s)$$

$$\int_{-\infty}^{\infty} e^{-ve} f(e)\, g(e - e_s) = f(v)\, g(v)$$

$$\tilde{\sigma}^{eik}(v) = \tilde{J}_1^{eik}(v)\, \tilde{J}_2^{eik}(v)\, S(v)$$

$$\sigma(v) = J_n(v)\, J_{\bar{n}}(v)\, S(v) = J_n(v)\, J_{\bar{n}}(v)\, \frac{\sigma_{eik}(v)}{J_1^{eik}(v)\, J_2^{eik}(v)}$$

soft function in SCET

soft function in pQCD

what's $\dfrac{J_n(v)}{J_1^{eik}(v)}$? in SCET?

abstract formula, concrete calcs. : Idilbi, Mehen

$$\mathcal{L}_c^q = \bar{\xi}_{\bar{n},p'} \left[i n\cdot D_{us} + g n\cdot A_{n,q} + (\perp) \right] \frac{\not{n}}{2} \xi_{n,p}$$

$$\xi_n = Y_n^\dagger \xi_n'$$

$$\mathcal{L}_c^{q'} = \bar{\xi}'_{\bar{n},p'} \left[i n\cdot \partial + \sum_{q\neq 0} g\, n\cdot A'_{n,q} + (\perp)' \right] \frac{\not{n}}{2} \xi'_{n,p}$$

$$\mathcal{L}_c^{q''} = \bar{\xi}''_{\bar{n},p'} \left[i n\cdot \partial + g n\cdot A''_{n,0} + \sum_{q\neq 0} g n\cdot A_{n,q} \right] \frac{\not{n}}{2} \xi''_{n,p}$$

$$\xi''_n = Z_n^\dagger \xi'_n$$

$$\mathcal{L}_c^{q'} = \bar{\xi}'_{\bar{n},p'} \left[i n\cdot \partial + \sum_{q\neq 0} g n\cdot A'_{n,q} \right] \frac{\not{n}}{2} \xi'_{n,p}$$

$$J_n^{\bullet}(e) \sim \sum_{N_{J_n}} |\langle N_{J_n} | \bar{\xi}''_n W''_n | 0\rangle|^2_{\mathcal{L}''} \, \delta(e - e(N_{J_n}))$$

$$= \sum_{N_{J_n}} |\langle N_{J_n} | \bar{\xi}'_n W'_n Z_n | 0\rangle|^2_{\mathcal{L}'} \, \delta(e - e(N_{J_n}))$$

$$W'_n = W_n^{\phi} \Omega_n$$

$$\Omega_n = P\exp\left[i g \int ds\, \bar{n}\cdot A_{n,0} \right]$$

$$= \sum |\langle N_{J_n}^{\phi} | \bar{\xi}'_n W_n^{\phi} | 0\rangle \langle N^{ek} | \Omega_n Z_n | 0\rangle|^2_{\mathcal{L}'} \, \delta(e - e_{\perp} - e_{N^{ek}})$$

$$= \int de_c \, J_n^{\phi}(e_c) \, J_n^{(ek)}(e - e_c)$$

$$J_n(v) = J_n^{\phi}(v) \, J_n^{ek}(v)$$

$$\sigma(v) = J_n^{\phi} J_{\bar{n}}^{\phi} \sigma^{ek} = \frac{J_n}{J_n^{ek}} \frac{J_{\bar{n}}}{J_{\bar{n}}^{ek}} \sigma^{ek}$$

III NP corrections from resummed pQCD

$$\frac{1}{\sigma_{tot}}\tilde{\sigma}_a(v) = 2\int_0^{Q^2}\frac{dp_\perp^2}{p_\perp^2}\,A(\alpha_s(p_\perp))\int_{p_\perp^2/Q^2}^{p_\perp/Q}\frac{du}{u}\left(e^{-u^{1-a}v(p_\perp/Q)^a}-1\right)$$

$$\int_0^{\mu_I^2} + \int_{\mu_I^2}^{Q^2} \qquad + \text{(B subleading p.c.'s)}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$S_a(v) \qquad\qquad \sigma_{PT}$$

Manohar
Becher, Neubert

IV Better factorization

$$\frac{d\sigma}{de} = \frac{1}{2Q^2}\sum_N |\langle N|\, j^\mu(0)\,|0\rangle L_\mu|^2\,(2\pi)^4\delta^4(q-p_N)\,\delta(e-e(N))$$

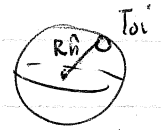
$$= \frac{1}{2Q^2}\frac{|L|^2}{3}\int d^4x\, e^{iq\cdot x}\,\langle 0|\, j_\mu^*(x)\,\delta\!\left(e-\frac{1}{a}\int d\eta\, f_e(\eta)\,\mathcal{E}(\eta)\right) j^\mu(0)\,|0\rangle$$

$$\mathcal{E}_T(\eta) \longleftrightarrow \mathcal{E}(\hat{n})\,|X\rangle = \sum_{i\in X} E_i\,\delta^2(\hat{n}-\hat{n}_i)\,|X\rangle$$

$$\delta^2(\hat{n}-\hat{n}_i) = \delta(\cos\theta-\cos\theta_i)\,\delta(\phi-\phi_i)$$

$$\mathcal{E}_T(\eta) = \frac{1}{\cosh^3\eta}\int_0^{2\pi} d\phi\,\mathcal{E}(\hat{n})$$

$$\mathcal{E}(\hat{n}) = \lim_{R\to\infty} R^2\int_0^\infty dt\,\hat{n}_i\,T_{0i}(t,R\hat{n})$$



$$T_{\mu\nu}^{SCET'} = T_{\mu\nu}^{c} + T_{\mu\nu}^{s}$$

$$\frac{d\sigma}{de} = \frac{|L|^2}{6Q^2}\int d^4x\, e^{iq\cdot x}\,\langle 0|\,\bar{\chi}_{\bar{n}}(x)\,\bar{Y}_{\bar{n}}^\dagger\gamma_\perp^\mu Y_n^\dagger\,\delta(e-F(T^c+T^s))\,\bar{Y}_{\bar{n}}\,\chi_{\bar{n}}\,|0\rangle$$

$$= \frac{|L|^2}{6Q^2}\int d^4x\, e^{iq\cdot x}\int de_J\,\langle 0|\,\bar{\chi}_n(x)\,\gamma_\perp^\mu\chi_{\bar{n}}(x)\,\delta(e_J-F(T^c))\,\chi_n(0)\,\gamma_\perp^\mu\,\bar{Y}_{\bar{n}}\,|0\rangle$$

$$\times\langle 0|\,\bar{Y}_{\bar{n}}^\dagger Y_n^\dagger(x)\,\delta(e-e_J-F(T^s))\,Y_n\,\bar{Y}_{\bar{n}}^\dagger(0)\,|0\rangle$$

guess $$= \int de_J\,\sigma_J(e_J)\,S(e-e_J)$$

question: how to calculate these matrix elements?

didn't discuss: $\frac{1}{Q^2}$ corrections to T_a distribution:

violations of universality $\Leftrightarrow$ correlations in soft radiation

SCET 08!