NNLO CORRECTIONS IN HADRONIC B DECAYS

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OUTLINE

$B \longrightarrow M_1 M_2$ DECAYS IN QCDF / SCET

CHALLENGING THE NNLO CALCULATION

- 1-LOOP SPECTATOR SCATTERING
- 2-LOOP VERTEX CORRECTIONS

COMPILATION OF NNLO RESULTS

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QCD Factorization

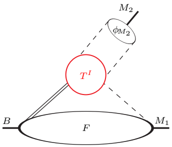
[Beneke, Buchalla, Neubert, Sachrajda 99,01]

$B \longrightarrow M_1 M_2$ reminder

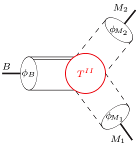
interest:	CKM angles, flavour mixing, CP violation, New Physics, ...
phenomenology:	many decay channels + observables, B factories, ...
main task:	quantitative control of hadron dynamics !

QCD Factorization

$\langle M_1 M_2 | Q_i | \bar{B} \rangle \ =$



$+$



$+ \, \mathcal{O}\left(\frac{1}{m_b}\right)$

F^{BM_1}	physical (not SCET) form factor	$T_i^I = \mathcal{O}(1)$	vertex corrections
ϕ_M	light-cone distribution amplitudes	$T_i^{II} = \mathcal{O}(\alpha_s)$	spectator scattering

$\longrightarrow$ strong phases small $\sim$ $\mathcal{O}(\alpha_s)$, $\mathcal{O}(1/m_b)$

Soft-Collinear Effective Theory

[Bauer, Fleming, Pirjol, Stewart; Beneke, Feldmann; Chay, Kim; Hill, Neubert, ... 01+]

$T_i^{I,II}$ as Wilson coefficients of non-local operators

$\rightarrow$ in particular $T_i^{II}(\omega, u, v) = \int dz \; H_i^{II}(u, z) \; J_{||}(z, \omega, v)$

H_i^{II} hard coefficient function ($QCD \rightarrow SCET_I$ at $\mu_h \sim m_b$)

$J_{||}$ (universal) jet function ($SCET_I \rightarrow SCET_{II}$ at $\mu_{hc} \sim \sqrt{\Lambda m_b}$)

resummation of $\ln \mu_h / \mu_{hc}$ (RGEqs in $SCET_I$ for $\mu_h > \mu > \mu_{hc}$)

M_2 factorizes already at hard scale $\mu_h \sim m_b$

$\rightarrow$ strong phases encoded in T_i^I and H_i^{II} only

further classification of power corrections

formulation of rigorous factorization proofs

...

Comparisons

[BBNS: Beneke, Buchalla, Neubert, Sachrajda 99+;
BPRS: Bauer, Pirjol, Rothstein, Stewart 04+;
ALRS: Arnesen, Ligeti, Rothstein, Stewart 06+]

QCDF and SCET . . .

. . . provide a systematic expansion of QCD for $m_b \to \infty$

. . . give the same theoretical predictions

. . . correspond to diagrammatical / effective theory approach

BBNS and BPRS . . .

. . . have different theoretical prejudices . . .

. . . about hard-collinear scale (perturbative / fit ζ_J to data)

. . . about charming penguins (short-distance / additional complex fit parameter)

BBNS and ALRS . . .

. . . differ conceptually in treatment of power-corrections
(model-dependent estimates $X_{H,A}$ / calculation with zero-bins) [Manohar, Stewart 06]

In the following $\longrightarrow$ QCDF/SCET analysis à la BBNS

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The NNLO challenge

Motivation

strong phases $\sim \mathcal{O}(\alpha_s)$	$\longrightarrow$	direct CP asymmetries known to LO only
cancellation in LO+NLO for α_2	$\longrightarrow$	enhancement from NNLO ?
spectator scattering with $\alpha_s(\mu_{hc})$	$\longrightarrow$	perturbation theory well-behaved ?
factorization proof incomplete	$\longrightarrow$	does factorization hold at all?
systematic framework	$\longrightarrow$	compute systematic corrections !

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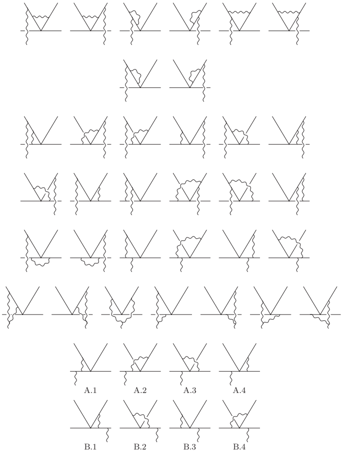
Available NNLO corrections

$J_{\|\|}$ matching + resummation	[Becher, Hill, Lee, Neubert 04; Becher, Hill 04; Kirilin 05; Beneke, Yang 05]	
H_i^{II} tree amplitudes	[Beneke, Jäger 05; Kivel 06; Pilipp 07prel]	new
penguin amplitudes	[Beneke, Jäger 06]	new
T_i^I tree amplitudes	[GB 06 (Im part)]	new
$\mathcal{O}(\alpha_s^2\beta_0)$-corrections	[Becher, Neubert, Pecjak 01; Burrell, Williamson 05]	

H_i^{II}: Tree amplitudes



Calculation

- $\mathcal{O}(50)$ 1-loop QCD diagrams
- $SCET_I$ side given by counterterms
- evanescent operators at tree level

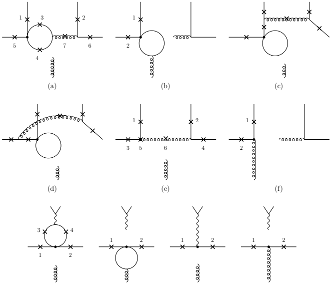
Main results [Beneke, Jäger 05; Kivel 06]

- factorization holds
- PT well-behaved at μ_{hc}
- Numerics ($\rightarrow$ later)

Interesting alternative [Pilipp 07prel]

- pure QCD calculation
- gives directly $T_i^{II} = H_i^{II} \otimes J_{||}$

H_i^{II}: Penguin amplitudes



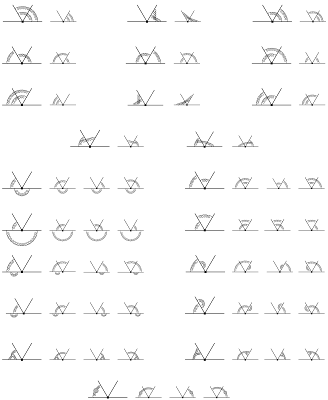
Calculation

- $\mathcal{O}(40)$ tree and 1-loop diagrams
- QCD and EW penguins
- technically easier than tree amps

Main results [Beneke, Jäger 06]

- factorization holds + PT well-behaved
- [Li, Yang 05] incomplete
- Numerics ($\longrightarrow$ later)

T_i^I: Tree amplitudes



Calculation

- essentially QCD calculation
- $\mathcal{O}(75)$ 2-loop diagrams
- evanescent operators at 1-loop

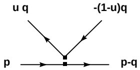
Main results [GB 06 (Im part)]

- factorization holds
- Numerics ($\rightarrow$ later)

2-loop vertex corrections: Overview

Characterization

+ T_i^I depend only on u (and m_c/m_b)
– 2-loop with 4 external lines (multi-loop + multi-leg)
$\longrightarrow$ first calculation of 2-loop matrix elements for $b \to uud$



Strategy

general tensor decomposition	$\longrightarrow$	$\sim$ 6.000 scalar integrals
automatized reduction algorithm	$\longrightarrow$	$\sim$ 30 Master Integrals
calculation of Master Integrals	$\longrightarrow$	main challenge
IR-structure $\sim 1/\varepsilon_{IR}^4$	$\longrightarrow$	calculate 5 coeffs of MIs

First step

focus on imaginary part ($\longrightarrow$ strong phases)

less diagrams, less MIs, 4 coeffs of MIs, NLO complexity, . . .

2-loop vertex corrections: Techniques

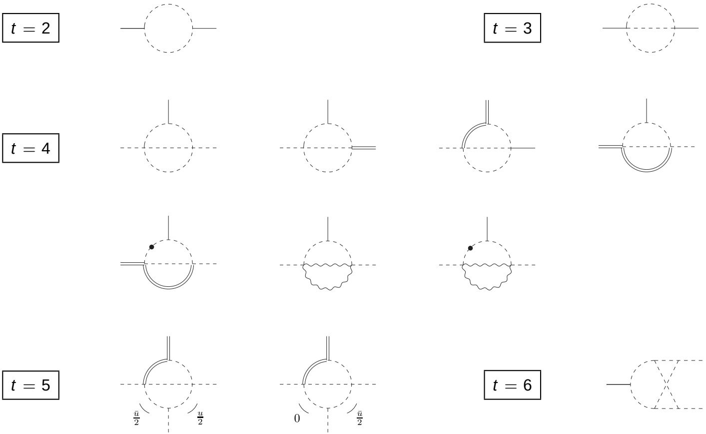
Reduction to MIs

- integration-by-parts identities [Chetyrkin, Tkachov 81]
- Lorentz-invariance identities [Gehrmann, Remiddi 99]
- solve system of $\mathcal{O}(10.000)$ equations efficiently [Laporta 00]

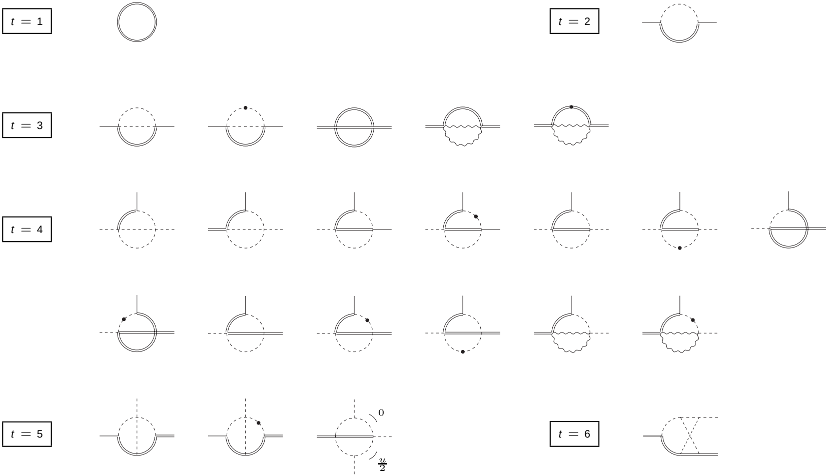
Calculation of MIs

- calculation with Feynman parameters much too difficult
- method of differential equations [Kotikov 91; Remiddi 97]
- Harmonic Polylogarithms [Remiddi, Vermaseren 99]
- Mellin-Barnes techniques for boundary conditions [Smirnov 99; Tausk 99]
- numerical check with method of sector decomposition [Binoth, Heinrich 00]

List of Master Integrals (Im part)



List of Master Integrals (Re part)



2-loop vertex corrections: Techniques

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IR-subtractions

$$\langle Q_i \rangle_{ren} \;=\; F\; T_i * \phi \qquad \text{UV finite, IR divergent}$$

$$\to\; F^{(0)}\, T_i^{(2)} * \phi^{(0)} \;+\; \underbrace{F^{(1)}}_{\sim \frac{1}{\varepsilon_{IR}^2}}\, T_i^{(1)} * \phi^{(0)} \;+\; F^{(0)}\, T_i^{(1)} * \underbrace{\phi^{(1)}}_{\sim \frac{1}{\varepsilon_{IR}}} \;+ \ldots$$

finite!

$\to$ cancellation of all UV- and IR-divergences provides important cross-check!

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Subtlety

need 1-loop kernels $T_i^{(1)}$ to $\mathcal{O}(\varepsilon^2)$

from $\quad \langle Q_i \rangle_{ren}^{(1)} \;=\; F^{(0)}\; T_i^{(1)} * \phi^{(0)}$

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Subtlety

need 1-loop kernels $T_i^{(1)}$ to $\mathcal{O}(\varepsilon^2)$

from $\quad \langle Q_i \rangle_{ren}^{(1)} \;=\; F^{(0)}\, T_i^{(1)} * \phi^{(0)} \;+\; \underbrace{F_E^{(0)}\, T_{i,E}^{(1)} * \phi_E^{(0)}}_{\mathcal{O}(\varepsilon)}$

$\rightarrow$ extend factorization formula to include evanescent structures

$\rightarrow$ evanescent 1-loop matrix elements $F_E^{(1)}$, $\phi_E^{(1)}$ contribute to physical $T_i^{(2)}$!

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Full NNLO result for Im$[\alpha_{1,2}]$

(preliminary)

default scenario

μ_h	μ_{hc}	m_c	f_B	$F_+^{B\pi}(0)$	$\lambda_B(1\text{ GeV})$	$a_2^\pi(1\text{ GeV})$
$4.8^{+4.8}_{-2.4}$	$1.5^{+0.9}_{-0.5}$	1.6 ± 0.2	0.21 ± 0.02	0.25 ± 0.05	0.48 ± 0.14	0.25 ± 0.20

$V^{(1)}$ [BBNS 01] $V^{(2)}$ [GB 06] $H^{(2)}$ [BJ 05] NNLO

$$\text{Im}[\alpha_1(\pi\pi)] = 0.012$$
$$\text{Im}[\alpha_2(\pi\pi)] = -0.077$$

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$V^{(1)}$	$V^{(2)}$	$H^{(2)}$	NNLO
[BBNS 01]	[GB 06]	[BJ 05]	

$$\text{Im}[\alpha_1(\pi\pi)] = 0.012 + 0.031$$

$$\text{Im}[\alpha_2(\pi\pi)] = -0.077 - 0.052$$

Full NNLO result for Im[$\alpha_{1,2}$] (preliminary)

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	$V^{(1)}$ [BBNS 01]		$V^{(2)}$ [GB 06]		$H^{(2)}$ [BJ 05]		NNLO
$\text{Im}[\alpha_1(\pi\pi)] =$	0.012	$+$	0.031	$-$	0.014	$=$	0.030 ± 0.020
$\text{Im}[\alpha_2(\pi\pi)] =$	-0.077	$-$	0.052	$+$	0.023	$=$	-0.107 ± 0.053

→ NNLO corrections important

→ partial cancellation between $V^{(2)}$ and $H^{(2)}$

G ∼ S4

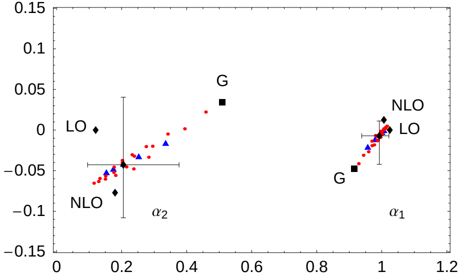
$$\text{Im}[\alpha_1] = 0.012 + 0.031 - 0.033$$
$$\text{Im}[\alpha_2] = -0.077 - 0.053 + 0.056$$

→ dominant uncertainties from μ_h, μ_{hc}, X_h

(RGI not yet included!)

Partial NNLO result for Re[$\alpha_{1,2}$]

[from Beneke/Jäger, hep-ph/0512351]



→ ∼ 20% enhancement of $C/T = \alpha_2/\alpha_1$

→ better description of $B \to \pi\pi$ data

Parameter set G ∼ S_4 $(\lambda_B = 0.2,\ a_2^\pi = 0.3)$

$10^6\,\mathrm{Br}(\bar{B}^0 \to \pi^0\pi^0) = 0.73^{+0.27}_{-0.24}(\mathrm{CKM})^{+0.52}_{-0.21}(\mathrm{hadr.})^{+0.35}_{-0.25}(\mathrm{pow.})$ $[\exp : 1.31 \pm 0.21]$

Partial NNLO result for penguins

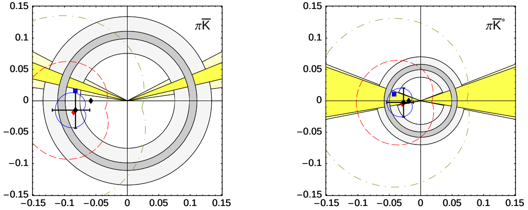
[from Beneke/Jäger, hep-ph/0610322]

Findings

(accidentally) small correction to QCD penguin amplitude α_4^p

sizeable contribution to EW colour-suppressed penguin amplitude $\alpha_{4,\mathrm{EW}}^p$

P/T-ratios



$$\frac{\hat{\alpha}_4^c(M_1M_2)}{\alpha_1(\pi\pi)+\alpha_2(\pi\pi)}$$

$\longrightarrow$ non-trivial check of the factorization framework!

Summary

on-going effort to calculate NNLO corrections in hadronic B decays

- $\longrightarrow$ 1-loop spectator scattering complete
- $\longrightarrow$ first results for 2-loop vertex corrections

factorization shown to hold in non-trivial order in PT

- $\longrightarrow$ IR-divergences cancel, convolutions are finite
- $\longrightarrow$ perturbation theory well-behaved at $\mu_h \sim m_b$ and $\mu_{hc} \sim \sqrt{\Lambda m_b}$

individual NNLO contributions can be sizeable

- $\longrightarrow$ partial NNLO results show somewhat better agreement with data
- $\longrightarrow$ full NNLO analysis might drastically change the pattern of CP asymmetries in QCDF/SCET